QFT on curved spacetimes: axiomatic framework and applications

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Plan of the course

Date	Topics
01.03.13	algebraic quantum mechanics, basic properties of C^*-algebras used in physics, **Proposed literature**: [3, 4, 14, 13, 16].
08.03.13	introduction to Lorentzian geometry, Haag-Kastler axioms, axioms of locally covariant quantum field theory (LCQFT). **Proposed literature**: [8, 2, 7].
15.03.13	remarks on the time-slice axiom and the Einstein causality axiom in LCQFT, **Proposed literature**: [8, 7, 1, 15].
22.03.13	locally covariant quantum fields, example of an LCQFT: Weyl algebra of a free scalar field, algebraic formulation of classical field theory, basics in microlocal analysis, deformation quantization. **Proposed literature**: [9, 10, 5, 6, 8].
27.03.13	interaction and time ordered products, introduction to Epstein-Glaser renormalization, renormalization group, example: ϕ^4 **Proposed literature**: [5].
03.04.13	Generalizations: gauge theories in the framework of algebraic quantum field theory **Proposed literature**: [11, 12].

References

[1] Ch. Bär, K. Fredenhagen *Quantum field theory on curved spacetimes*, Lecture Notes in Physics, Vol. 786, Springer Berlin Heidelberg 2009.

[2] Ch. Bär, N. Ginoux, F. Pfäffle *Wave Equations on Lorentzian Manifolds and Quantization*, ESI Lectures in Mathematics and Physics, European Mathematical Society Publishing House 2007.

[3] Bratteli, O., Robinson, D.W., *Operator algebras and quantum statistical mechanics, vol. 1*, 2nd edn., Springer-Verlag, Berlin, 1987.

[4] Bratteli, O., Robinson, D.W., *Operator algebras and quantum statistical mechanics, vol. 2*, 2nd edn., Springer-Verlag, Berlin, 1997.

[5] R. Brunetti, M. Dütsch, K. Fredenhagen, *Perturbative Algebraic Quantum Field Theory and the Renormalization Groups*, Adv. Theor. Math. Phys. **13** Number 5 (2009) 1541-1599, [arXiv:math-ph/0901.2038v2].

[6] R. Brunetti, K. Fredenhagen, *Microlocal Analysis and Interacting Quantum Field Theories: Renormalization on Physical Backgrounds*, Commun. Math. Phys. **208** (2000) 623-661.

[7] R. Brunetti, K. Fredenhagen, P. Imani, K. Rejzner *The Locality Axiom in Quantum Field Theory and Tensor Products of C^*-algebras*, [arXiv:math-ph/1206.5484].

[8] R. Brunetti, K. Fredenhagen, R. Verch, *The generally covariant locality principle - A new paradigm for local quantum field theory*, Commun. Math. Phys. **237** (2003) 31-68.

[9] M. Dütsch and K. Fredenhagen, *Perturbative algebraic field theory, and deformation quantization*, Proceedings of the Conference on Mathematical Physics in Mathematics and Physics, Siena June 20-25 2000, [arXiv:hep-th/0101079].

[10] M. Dütsch and K. Fredenhagen, *Causal perturbation theory in terms of retarded products, and a proof of the action Ward identity*, Rev. Math. Phys. **16** Issue 10 (2004) 1291-1348, [arXiv:hep-th/0403213].

[11] K. Fredenhagen, K. Rejzner, *Batalin-Vilkovisky formalism in the functional approach to classical field theory*, [arXiv:math-ph/1101.5112].

[12] K. Fredenhagen, K. Rejzner, *Batalin-Vilkovisky formalism in perturbative algebraic quantum field theory*, [arXiv:math-ph/1110.5232].

[13] R. Haag, *Local Quantum Physics*, 2nd ed.

[14] R.V. Kadison, J.R. Ringrose, *Fundamentals of the Theory of Operator Algebras.*

[15] L. Hörmander *The analysis of linear partial differential operators I: Distribution theory and Fourier analysis*, Springer 2003.

[16] B. Simon, M. Reed, *Methods of Modern Mathematical Physics, I: Functional Analysis*, Academic Press, 1980.

â€¢ To regulate the QFT, we truncate the Hilbert space by expanding in a finite field basis on each simplex called a finite element basis. ð“. ðœ™ðœŽ ðœ‰ = à· ð¸ð‘– ðœ‰ ðœ™ð‘–.Â â€¢ General proof of renormalizability on curved lattice is hard. â€¢ No translation symmetry, no Fourier techniques â€¢ No closed form for the propagator at finite lattice spacing. â€¢ Nonetheless, we propose a scheme which follows the spirit of the perturbative renormalization scheme of Reitz. It is important to note that QFT in curved spacetime is not quantum gravity. In a theory of quantum gravity we would take the backreaction of quantum effects on the spactime into account. In contrast, in curved spacetime QFT the curved spacetime is merely a background structure, i.e. an external source and not treated quantum mechanically. Concrete. The standard reference is Quantum Fields in Curved Space by N.D. Birrell, P.C.W. Davies. Abstract. The motto in this section is: the higher the level of abstraction, the better. For research of QFT in curved spacetime there are actually two quite different approaches, Lagrangian QFT and axiomatic QFT. I don't know much about the former, I only skimmed this recently published texbook: Leonard Parker, David Toms: "Quantum Field Theory in Curved Spacetime: Quantized Fields and Gravity", (Cambridge Monographs on Mathematical Physics). So at least two people are still active in that area :-) People in axiomatic QFT have been busy to generalize the Haag-Kastler axioms to curved spacetimes. There are several concepts that obviously have to be generalized, like the spectral condition that uses the Poincare group. This has been accomplished several years ago and now people are generalizing theorems and concepts.