GAUGE THEORIES OF GRAVITATION

A Reader with Commentaries

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Foreword by
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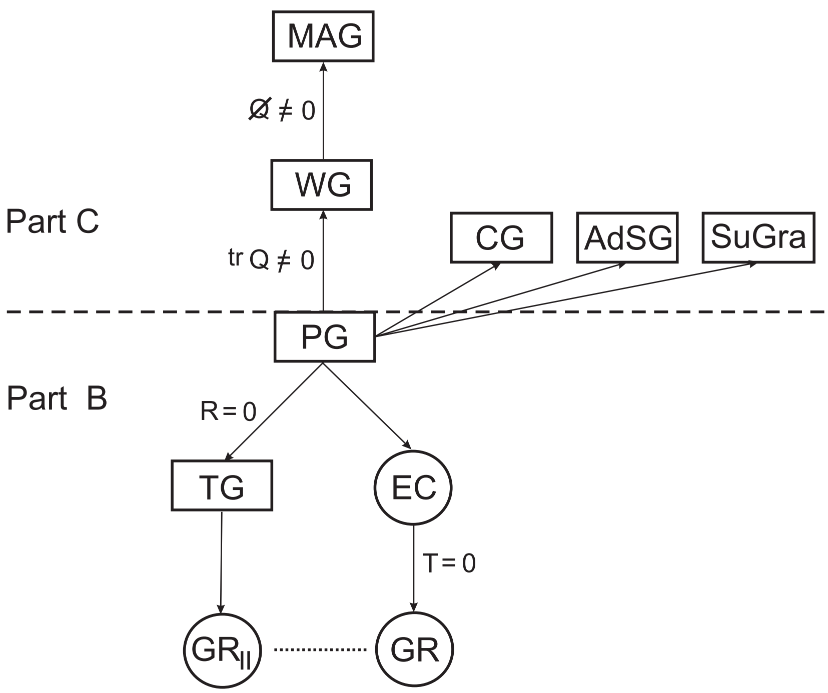
Classification of gauge theories of gravity



PG = Poincaré gauge theory (of gravity), **EC** = Einstein–Cartan(–Sciama–Kibble) theory (of gravity), **GR** = general relativity (Einstein's theory of gravity), **TG** = translation gauge theory (of gravity) aka teleparallel theory (of gravity), $\mathbf{GR}_{||}$ = a specific TG known as teleparallel equivalent of GR (spoken "GR teleparallel"), **WG** = Weyl(–Cartan) gauge theory (of gravity), **MAG** = metric-affine gauge theory (of gravity), **CG** = conformal gauge theory (of gravity), **AdSG** = (anti-)de Sitter gauge theory (of gravity), **SuGra** = supergravity (super-Poincaré gauge theory of gravity).

The symbols in the figure have the following meaning: rectangle $\square \rightarrow$ class of theories; circle $\circ \rightarrow$ definite viable theories; nonmetricity $Q = \not{Q} + \frac{1}{4}(\mathrm{tr}\,Q)1$, torsion T, curvature R.

Contents

19.3 C. W. F. Everitt, Gravity Probe B: I. The scientific implications, in: *Sixth Marcel Grossmann Meeting on General Relativity, Part B* (World Scientific, Singapore, 1992), pp. 1632–1644; extract

19.4 J.-P. Hsu and D. Fine (eds.), *100 Years of Gravity and Accelerated Frames, The Deepest Insights of Einstein and Yang–Mills* (World Scientific, Hackensack, NJ, 2005); extract

19.5 W.-T. Ni, Yang's gravitational field equations, *Phys. Rev. Lett.* **35**, 319–320 (1975)

Foreword

Symmetry has always played a big role in physics. Advancing understanding has time and again revealed previously unknown symmetries. Isaac Newton abandoned the idea of a preferred origin of space, revealing the underlying translational symmetry; Albert Einstein uncovered an unexpected symmetry between time and space.

A key innovation of the twentieth century was Hermann Weyl's invention of gauge theory, in which a global physical symmetry is replaced by a local one; the arbitrary phase in the quantum wave-function becomes a function of space and time, a change that requires the existence of the electromagnetic field. This proved to be an astonishingly fruitful idea. Today, all the components of the "standard model" of particle physics that so accurately describes our observations are gauge theories. Weyl's "gauge principle", that global symmetries should be promoted to local ones, applied to the standard-model symmetry group $SU(3) \times SU(2) \times U(1)$, is enough to yield the strong, weak and electromagnetic interactions.

Only gravity is missing from this model. But it too shows many of the same features. Going from special to general relativity involves replacing the rigid symmetries of the Poincaré group—translations and Lorentz transformations—by freer, spacetime dependent symmetries. So it was natural to ask whether gravity too could not be described as a gauge theory. Is it possible that starting from a theory with rigid symmetries and applying the gauge principle, we can recover the gravitational field? The answer turned out to be yes, though in a subtly different way and with an intriguing twist. Starting from special relativity and applying the gauge principle to its Poincaré-group symmetries leads most directly not precisely to Einstein's general relativity, but to a variant, originally proposed by Élie Cartan, which instead of a pure Riemannian spacetime uses a spacetime with torsion. In general relativity, curvature is sourced by energy and momentum. In the Poincaré gauge theory, in its basic version, there is also torsion, sourced by spin.

As someone who was involved in the early stages of this development, I am astonished and intrigued by how the theory has developed over the last half century. Reading this book makes it clear how wide its ramifications have spread. Over the years, Poincaré gauge theory has been put on a much firmer mathematical base. In its simplest form, it gives predictions that are in almost all observational situations identical with those of general relativity, but in situations of extremely high density there are significant differences. These differences may be of profound importance for the physics of the very early universe and of black holes, and could one day be subject to observational test.

Moreover, Poincaré gauge theory is not necessarily the end of the story. There are several possible extensions, in which the basic symmetry group is even larger; the Poincaré group may be augmented by the inclusion of dilatations or even enlarged to the full group of affine transformations. The resulting theories, the Weyl–Cartan theory and the metric-affine gravity theory, have some very attractive features. Only time will tell whether any of these intriguing theories is correct and which of the hypothesized hidden symmetries is actually realized in nature. For anyone interested is pursuing these ideas, this book certainly provides a fascinating and very valuable resource.

London, March 2012

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Preface

We have been both fascinated by gauge theories of gravity since the 1960s and the 1970s and have followed the subject closely through our own work. In this reprint volume with commentaries we would like to pass over our experience to the next generation of physicists. We have tried to collect the established results and thus hope to prevent double work and to focus new investigations on the real loopholes of the theory.

The aim of this reprint volume with commentaries is to introduce graduate students of theoretical physics, mathematical physics or applied mathematics, or any other interested researcher, to the field of *classical gauge theories of gravity*. We assume that our readers are familiar with the basic aspects of classical mechanics, classical electrodynamics, special relativity (SR), and possibly elements of general relativity (GR). Some knowledge of particle physics, group theory, and differential geometry would be helpful.

Why gauge theory of gravity? Because all the other fundamental interactions (electroweak and strong) are described successfully by gauge theories (of internal symmetries), whereas the established gravitational theory, Einstein's GR, seems to be outside this general framework, even though, historically, the roots of gauge theory grew out of a careful analysis of GR. A full clarification of the gauge dynamics of gravity might be the last missing link to the hidden structure of a consistent unification of all the fundamental interactions at both the classical and the quantum level.

Our book is intended not just to be a simple reprint volume, but more a guide to the literature on gauge theories of gravity. The reader is expected first to study our introductory commentaries and become familiar with the basic ideas, then to read specific reprints, and after that to return again to our text, explore the additional literature, etc. The interaction is expected to be more complex than just starting with commentaries and ending with reprints. A student, guided by our commentaries, can get self-study insight into gauge theories of gravity within a relatively short period of time.

The underlying structure of gravitational gauge theory is the group of motions of the spacetime in SR, namely the Poincaré group $P(1,3)$. If one applies the gauge-theoretical ideas to $P(1,3)$, one arrives at the Poincaré gauge theory of gravity (PG). Therein, the conserved energy-momentum current of matter and the spin part of the conserved angular momentum current of matter both act as sources of gravity. The simplest PG is the

Einstein–Cartan theory, a viable theory of gravity that, like GR, describes all classical experiments successfully. On the other hand, if one restricts attention to the translation subgroup of $P(1,3)$, one ends up with the class of translation gauge theories of gravity, one of which, for spinless matter, can be shown to be equivalent to GR. The developments that led to PG are presented in Part A of our book; in Part B, definite and enduring results of PG are displayed. The content of Parts A and B should be considered as a mandatory piece of the general education for *all gravitational physicists*, while the remaining two parts cover subjects of a more specialized nature.

Since SR is such a well-established theory, from a theoretical as well as from an experimental point of view, the gauging of $P(1,3)$ rests on a very solid basis. Nevertheless, there arise arguments as to why an extension of PG seems desirable; they are presented in Part C. As a finger exercise, we gauge the group of Poincaré plus scale transformations. Then, we extend $P(1,3)$ to the general real linear group $GL(4,R)$, thus arriving at metric-affine gauge theory of gravity (MAG). This general framework leads to a full understanding of the role of a non-vanishing gradient of the metric (nonmetricity). Several other extensions treated in Part C appear to be rather straightforward tasks.

The gauge theory of gravity, since 1961, when it first had been definitely established, has had a broad development. Therefore, in Part D we display the results on several specific aspects of the theory, like the Hamiltonian structure, equations of motion for matter, cosmological models, exact solutions, three-dimensional gravity with torsion, etc. These subjects could be starting points for research projects for our prospective readers.

Clearly, making a good choice of reprints is a very demanding task, particularly if we want to take care of the historical justice and authenticity. But we also wanted to take care of another aspect—that our collection of reprints should be a useful guide to research-oriented readers without too many historical detours. These two aspects are not always compatible, and we tried to ensure a reasonable balance between them. To what extent these attempts were successful is to be judged by our readers.

• Chapters of the book that can be skipped at a first reading are marked by the star symbol *.

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We thank Professor Kibble, one of the founders of the gauge theory of gravity, who honored us by writing a foreword to this book.

List of useful books

Here is a chronologically ordered list of books, in which readers can find useful information on the subject of gauge theories of gravity. The selection is made by requiring at least some mentioning of the EC theory.

- V. N. Ponomariev, A. O. Barvinsky, and Yu. N. Obukhov, *Geometrodynamical Methods and the Gauge Approach to the Theory of Gravitational Interactions* (Energoatomizdat, Moscow, 1985) (in Russian)
- W. Thirring, *A Course in Mathematical Physics 2: Classical Field Theory, 2nd ed.*, translated by E. M. Harrell (Springer, New York, 1986)
- E. W. Mielke, *Geometrodynamics of Gauge Fields—On the Geometry of Yang–Mills and Gravitational Gauge Theories* (Akademie-Verlag, Berlin, 1987)
- M. Göckeler and T. Schücker, *Differential Geometry, Gauge Theories and Gravity* (Cambridge University Press, Cambridge, 1987)
- P. Ramond, *Field Theory: A Modern Primer, 2nd ed.* (Addison–Wesley, Redmond City, CA, 1989)
- W. Kopczyński and A. Trautman, *Spacetime and Gravitation* (PWN, Warsaw; Wiley, Chichester, 1992)
- M. Blagojević, *Gravitation and Gauge Symmetries* (IoP, Bristol, 2002)
- T. Ortín, *Gravity and Strings* (Cambridge University Press, Cambridge, 2004)
- L. Ryder, *Introduction to General Relativity* (Cambridge University Press, Cambridge, 2009)

Gravitation and Gauge Symmetries (Series in High Energy Physics, Cosmology and Gravitation). M Blagojevic. 5.0 out of 5 stars 2.Â This is not just an ordinary reprint volume; it is a guide to the literature on gauge theories of gravity. The reader is encouraged first to study the introductory commentaries and to become familiar with the basic content of the reprints and related ideas, then he/she can choose to read a specific reprint or reprints, and after that he/she should return again to the text and explore the additional literature, etc. Gauge-theoretic formalism (universal principle of the local invariance and the mechanism of spontaneous breaking of the gauge symmetry) forms the basis for the modern understanding of fundamental physical interactions and is successfully confirmed by the experimental discoveries of the gauge bosons and the Higgs particle. The carefully selected material of the book provides a minimal but sufficient mathematical introduction to the methods of the gauge gravitational theory, and gives a concise but exhaustive description of all specific physical consequences. In order to describe the... To construct gauge gravitation theory, we are based on the fact that Dirac fermion elds possess only Lorentz symmetries and therefore imply breaking of world symmetries. This symmetry breaking can occur i a pseudo-Riemannian metric ex-ists on a world manifold X [9, 10]. This metric h denes the representation of the holonomic coframes {dxÎ»} in the cotangent bundle T âˆ—X by the Dirac Î³-matrices.Â In gauge gravitation theory as like as in other gauge models, symmetry breaking. is formalized by the mathematical condition that there exists a principal subbundle. LhX of the frame bundle LX whose structure group is the Lorentz group L =. Towards a gauge theory of gravity. We saw: to describe electrodynamics as a gauge theory, we have to 1. forget about electrodynamics (!) 2. carry out a gauge procedure with a suitable group, here: U(1) 3. obtain electrodynamics for free from gauge curvature Lagrangian. To describe gravity as a gauge theory, we first have to forget about gravity. What remains if we do that? special relativity, and fields propagating on flat Minkowski space Note the difference: symmetries in external space, not in internal space. 8/11. Symmetries of Minkowski space.Â M. Blagojevic and F. W. Hehl (eds.), â€œGauge Theories of Gravitation â€“ A reader with commentaries,â€ (Imperial College Press, London, 2013). 11/11. Gauge Theories of Gravitation. Item Preview. > remove-circle.Â During the last five decades, gravity, as one of the fundamental forces of nature, has been formulated as a gauge field theory of the Weyl-Cartan-Yang-Mills type. The resulting theory, the Poincar\'e gauge theory of gravity, encompasses Einstein's gravitational theory as well as the teleparallel theory of gravity as subcases. In general, the spacetime structure is enriched by Cartan's torsion and the new theory can accommodate fermionic matter and its spin in a perfectly natural way. The present reprint volume contains articles from the most prominent proponents of the theory and is supplemented by detailed commentaries of the editors.