Norma Boakes, The Richard Stockton College of New Jersey

Origami-Mathematics Lessons: Paper Folding as a Teaching Tool

Spatial abilities are an important skill that children must develop as they study mathematics. This article looks at the art of Origami as a method for developing spatial skills as well as children's mathematical knowledge, disposition, and attitude about math. The author discusses how the use of Origami-mathematics lessons implemented into a geometry unit impacted students' math and spatial skills. Research findings and general attitudes of participants are shared providing support for this approach. Readers also learn how to create their own origami-mathematics lesson for use in their math classroom.

Introduction

Origami is not an uncommon practice in the mathematics classroom. A variety of publications from books like *Math in Motion: Origami in the Math Classroom* (Pearl, 1993) and articles such as Robichaux and Rodrigue's *Using Origami to Promote Geometric Communication* (2003) exist proposing the art of paperfolding as an effective mathematics teaching tool. While there is little argument that mathematics educators have used Origami in a variety of ways for instruction, there is a surprising lack of research investigating such claims (Boakes, 2006). In this article, the author will report on research seeking to quantify Origami's impact on spatial visualization and its affect on mathematical ability. Information presented here will also establish why a mathematics teacher would want to consider teaching Origami-math lessons of his or her own and how to go about getting started.

The Importance of Spatial Visualization

In *Principles and Standards for School Mathematics* [henceforth the Standards], spatial visualization is defined as "building and manipulating mental representations of two- and three-dimensional objects and perceiving an object from different perspectives" (NCTM, 2000, p. 41). Identified as an integral part of the K-12 geometry curriculum, National Council of Teachers of Mathematics (NCTM) emphasizes that children be given the opportunity to explore and develop their understanding of shapes and structures through engaging hands-on activities as a way to build their spatial skills. From an everyday perspective, spatial skills can benefit children, assisting them with such spatial tasks as navigating a map, drawing a floor plan, creating artwork, and determining a route to travel (Bishop, 1997; NCTM, 2000). Students additionally build their logical reasoning skills and ability to describe their physical environment. While this is true, research on the learning of mathematics reports "elementary and middle school students in the United States are failing to learn basic geometric concepts and geometric problem solving" as well as "woefully unprepared" for more advanced concepts taught at the high school level (Clements & Battista, 1992). Though improvement has been found through measures such as the National Assessment of Educational Progress over more recent years, there remains much work to be done in the area of geometry instruction.

Focusing specifically on the middle grades, the Standards describe the study of geometry as “thinking and doing” (NCTM, 2000, p. 165). Children need to explore geometric shapes and objects first hand, allowing them to develop their own understanding of geometric relationships. Whether it is building a three-dimensional model, drawing two-dimensional shapes, or working with manipulatives, it is this active play that enhances children’s spatial skills. Recommending a variety of tools, the Standards specifically support activities in which students use two-dimensional shapes to make three-dimensional shapes and deconstruct three-dimensional shapes into their two-dimensional equivalents (NCTM, 2000). It is through this type of activity that students develop their spatial visualization. A clear connection to this, and one commonly recognized by mathematics teachers at all levels, is the use of the art of paper folding, Origami, as a teaching tool. Origami, by nature, involves the learner in following a construction process moving a two-dimensional square into a variety of three-dimensional shapes and figures.

Investigating Origami’s Impact on a Group of Students

To determine how the instruction of Origami would impact student spatial visualization and the student’s learning of geometry, a study was conducted within a middle school teacher’s mathematics classes. One group of children received their normal geometry instruction based on the teacher’s set curriculum while the other received the same instruction in addition to what has been termed “Origami-mathematics” lessons (Boakes, 2006). A month long unit in geometry served as the time frame with Origami-mathematics lessons infused regularly within the teacher’s instruction. To determine how the students’ were impacted by the instruction pre- and post-tests to assess both spatial visualization abilities and mathematics achievement were used. Results from this study revealed both methods of instruction to be equally beneficial in terms of mathematical ability (Boakes, in press). As far as spatial ability, while students showed similar gains in the area of blended visualization (both two- and three-dimensional), there was evidence that males and females differed in how Origami affected their two-dimensional manipulation skills. Adjusted gains in scores showed males experiencing Origami performing better than males with only traditional instruction. In contrast, females reached higher gains with traditional instruction while those experiencing Origami gains were reduced. Though it cannot be said for certain why this was the case, factors contributing to this result may have included: students’ visual-spatial experiences outside of school such as video games; differences in the students’ confidence, males often more confident than their female counterparts; and the type of assessment tool selected with the test in this case shown to favor males more often than females (Boakes, 2006). Even with the slight differences shown among genders, students seemed to benefit from the change in instruction as a whole. Thus, this study lends support to the many claims that Origami is a beneficial method of instruction.

Teaching an Origami-mathematics lesson

As one might expect, Origami-mathematics lessons combine mathematics with the teaching of this ancient craft. Defined by Boakes, such a mathematics lesson involves “[teaching] using an origami activity linking students’ mathematics knowledge and skill during the folding process and with the resultant Origami figure” (2006, p.32). Similar to methods shared by Robichaux and Rodrigue (2003) as well as Cipoletti and Wilson (2004) in recent NCTM publications, the concept is to blend mathematical vocabulary and content within the steps an instructor must go through to teach the folding of a particular Origami model. Robichaux and Rodrigue's focus was on fostering geometric communication through discussion and investigation of completed Origami models. Cipoletti and Wilson sought to adapt Origami instructions by blending in naturally occurring mathematics terms and concepts. The Origami-mathematics lesson is a blend of both of these methodologies. Referring to Figure 1, the lesson is designed around the model to be folded, in this case a simple sailboat.

<u>Lesson #1- Sailboat Model</u>

Math Concepts:
Shape, area, parallel and perpendicular, spatial relations

Math Vocabulary:
Parallel lines
Right triangle
Perpendicular lines
Area
Angles- acute, obtuse, right
Quadrilateral
Trapezoid

Key Questions:

1. When you make both folds what shapes do you make? [*Squares*] How do the areas of the new squares compare to the old ones? [*They're one-fourth of the original square.*] What about the fold lines, do you recognize them? [*Yes, there perpendicular lines*.] How do you know? [*They meet at right angles*.]

2. When you fold in the diagonal of the square, what kind of shapes do you have now? [*Squares and right triangles*.] Where do all the fold lines meet? [*At the midpoint of the segments.*] What kind of angles can you find if you darken in the line segments? [*Have students show where acute, obtuse, and right angles formed.*]

3. What kind of shapes do you see once you fold the corners in? [*Right triangles and squares again.*] Can you find parallel or perpendicular lines anywhere? [*Have students show where they are on model.*]

4. Once you squash-fold your model, what shapes do you find? [*Right triangles.*] How does the area of the red triangle compare to the two smaller white ones? [*It's twice as big.* Can you still find parallel or perpendicular line segments? [*Have students show where they are.*]

5. With the last fold done, what shape is the base of the boat? [*Quadrilateral.*] Does it have a special name? [*Trapezoid.*]

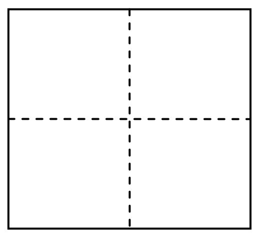
(Boakes, 2006, p. 177)

Figure 1. A sample Origami-mathematics lesson

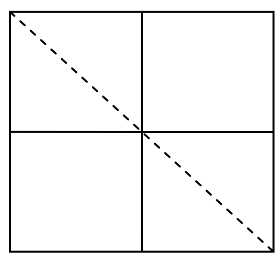
(See Figure 2 for folding instructions.) First, vocabulary is targeted that relates to the different geometric figures and shapes produced within the folding steps of the model. Then, a dialogue is designed to accompany each step. Shown in normal type are the questions the teacher poses after the folding step is complete, causing students to reflect on what they've created thus far. The possible responses by students that the teacher might expect follow in italics. While the actual responses the teacher may get could vary, this is the kind of rich discussion Origami instruction can foster. Effects that have been observed by the author include seeing an improvement in participation, more mathematical vocabulary interspersed in student dialogue, a gradual improvement in visualizing each folding step, and a heightened level of interest overall.

<u>Folding instructions</u>

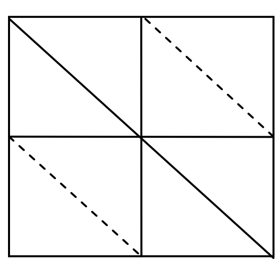
1. Start with white side up. Fold one side of square to meet its opposite side. Do this for both set of opposite sides.



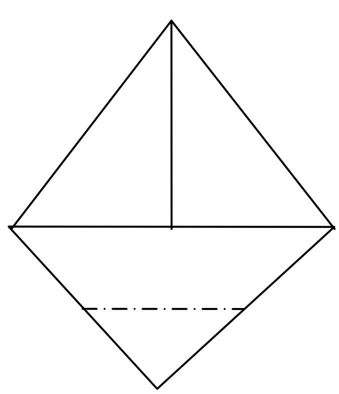
2. Flip paper over to colored side. Then fold one corner of the square to meet an opposite corner of a square. Crease along fold line then open back up to the original square.

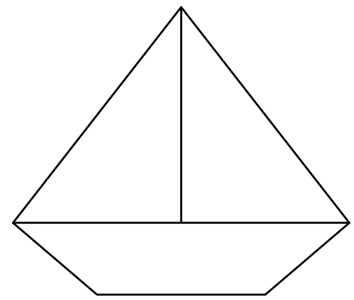


3. Flip back to the white side. Take one set of opposite corners and fold them into the center of the square, where the two existing fold lines meet.



4. Hold paper in a cupped hand so it sits naturally in it. Gently push finger in center of square where folds all meet at a point. Carefully squash along folds to make shape shown.





5. Fold red corner up to meet the point where the two white triangles meet.

Figure 2. Illustrated directions for the Sailboat model.

Creating your own Origami-mathematics lesson

Creating an Origami-mathematics lesson is a simple process that teachers of any grade level can do. The first step is to seek out an Origami model appropriate for the age and grade of the students being taught. Sources for these are easy to come by. Libraries, local bookstores, even yearly calendars all offer excellent choices. Shown below in Figure 3 are a few recommended books to get started.

Math in Motion: Origami in the Classroom (K-8) by B. Pearl

Unfolding Mathematics with Unit Origami by B. Franco

The First Book of Origami by Kodansha International

Origami Toys by F. Temko

Origami Math: Grades 4-6 by K. Baicker

Note: *All resources listed were used when designing Origami-mathematics lessons for the research study by Norma Boakes.*

Figure 3. Great Books for Getting Started.

The Internet also offers a multitude of websites dedicated to Origami. Figure 4 below lists websites that may be useful in getting started.

Name of Site	Site Content
Eric's Origami Page	**This site has a whole section dedicated to Origami and its connection to mathematics. There are also links to Origami history, various Origami models, and other sites you can go to.** ***www.paperfolding.com***
Origami Club	**Outstanding starter website with animated instructions. Models are categorized and easy to access. One of the easiest sites to navigate and use with children.** ***www.origami-club.com/en/***
Oriland	**A very user friendly site including a variety of links including step by step illustrated Origami instructions and an extensive gallery of Origami art.** ***www.oriland.com***
Robert J. Lang Origami	**Known as a "guru" of Origami art, this site is a must see. His work tends to be very complex but shows the artistic side of what many see just as a craft or hobby.** ***www.langorigami.com***
Sarah's Origami	**A great site for finding good models appropriate for all ages. Models are categorized and instructions are well drawn.** ***www.sarahsorigami.com***
Tammy Yee's Origami Page	**A wonderful site designed for beginners. Designs are very easy to follow and many have fun printable paper to make the final Origami model look real!** ***www.tammyyee.com/origami.html***

Figure 4. Useful Origami websites.

While searching, the teacher should consider what the purpose of the model will be. Perhaps it's to review terminology covered over a long period of time. It could be to highlight a specific geometric object currently being studied. In general, models can be found for almost every geometric topic one might need. Another consideration is the difficulty level of the model. The teacher should seek out books that identify the level of the folder and that offer simply illustrated steps.

With the model selected, it's time to prepare the Origami-mathematics lesson. The very first step is for the teacher to fold the model. While doing so, the teacher should consider the different geometric concepts and terms that might apply, focusing on those related to the goals of the lesson. Once the model is complete, compile a list of vocabulary that will be infused within the questions to be posed to students. The final step is to match the vocabulary, questions to be asked, and the illustrated instructions for folding the Origami model. The easiest way to accomplish this is for the teacher to do one step at a time, each time pausing to write the questions he or she wishes to pose to the students. Once questions are written, the teacher should brainstorm and record the types of responses he or she hopes to get. Once all questions and responses for each step have been recorded, the teacher should reflect once more on the resultant figure. Questions can also be written by the teacher to highlight concepts at this level. For instance, this may be a perfect time to discuss three-dimensional figures and their attributes. With the vocabulary listed, the questions and responses scripted, and the Origami model instructions located the teacher is ready to share it with his or her students. (See Figure 5 below for a synopsis of the steps to follow in creating your own Origami-mathematics lesson.)

Creating your own Origami-Mathematics lesson
1. Seek out an Origami publication that fits you and your students needs. (i.e. level of difficulty, illustration quality)
2. Consider what mathematics concepts and/or vocabulary you wish to highlight as you choose your Origami model.
3. Go through the folding process of your Origami model. While doing so list vocabulary that matches with your goals set.
4. Fold the model again. At each step stop and write your teacher questions being mindful of your vocabulary list.
5. Answer your own teacher questions and record the kind of answers you expect to hear.

Figure 5. Steps in designing an Origami-mathematics lesson.

Tips for Newcomers

Many people fear Origami if they have not attempted it before or have had little success with it in the past. For the teacher that has never folded before, it is beneficial to spend some time familiarizing oneself with the symbols and illustrations of Origami instructions. The best way to foster this process is to seek out a few beginner level Origami books. Ones targeting children are particularly useful. Once the teacher has practiced a few models, he or she often finds he or she is much better than he or she anticipated and has learned how to decode Origami symbols. If fortunate, some areas actually have local Origami groups where teachers can attend hands-on training sessions. (An excellent place to look for these is to contact Origami-USA, a national non-profit Origami association, consisting of Origami enthusiasts all over the US.)

Before bringing any Origami lesson to the classroom, the teacher should be reminded that children are very much like the adults that teach them. They may never have heard of the craft nor have had experience with it. The teacher, therefore, should take care to introduce students to the art. Provide students with a brief history of the art. Discussing the process the folder goes through and providing them with some basic Origami terminology also goes a long way in reducing any anxiety that might be present among children. The teacher should invest time in this process prior to using an Origami-mathematics lesson to avoid potential resistance and to build students' confidence. A final motivator is of course for the teacher to share what the students' model will look like when complete. If the teacher has access to previously done models, he or she should share these as well to spark interest in the art.

What Others Had to Say about Origami-Mathematics Lessons

Boakes, the creator of Origami-mathematics lessons, has had a great deal of experiences with Origami and its infusion into the mathematics curriculum. Having presented several workshops on Origami at both regional and national NCTM conferences, one thing is abundantly clear. Mathematics teachers love using Origami. Many say they already use it and want to learn more models. Others have never seen it but are in need of more hands-on motivational teaching techniques. There are still those who are simply curious about the art and the colorful models that can be produced in a variety of shapes and sizes. There is consensus among participants that Origami as a teaching tool can benefit students in a variety of ways when blended within mathematics instruction.

Curious to see how students truly felt about doing Origami in their classroom, Boakes had students who had done Origami-mathematics on a regular basis respond to a brief survey. Results were amazingly positive. Students reported their enjoyment the experience, felt it helped them understand the geometry they learned, and even thought it would be helpful in other areas of mathematics. When asked to describe their experiences with one word, students' responses included the following descriptors: helpful, exciting, interesting, easy, joyful, good job, enjoyable, awesome, fantastic, fine, and fun-filled. Positive comments were also common among participants such as:

> "It was fun learning how to do it."
>
> "I thought making the angles and discussing the lines of symmetry and such was a great and fun way of learning. Bringing all math topics into paper folding should be an actual activity in everyday math."
>
> "I thought this was so cool to do Origami. It was fun and educational."
>
> "I think that it was fun cool. I really liked it. It helped you learn funner and easier."
>
> "I thought it was fun and at the same time I was learning."

In all, Boakes found students to be very positive about their experience and that they felt it helped them to grasp the geometry concepts and terms they were learning.

Many that have written about Origami in the classroom have uncovered similar results. Robichaux and Rodrigue (2003) shared how fun students found Origami lessons in a middle school classroom. Students were said to be engaged, motivated, and excited to do paper-folding. The prevalence of math anxiety is mentioned by Barbara Pearl (1993) while discussing her book on Origami, *Math in Motion*. She alludes to Origami as a way to reduce students' fears of math and increase students' positive experiences in the math classroom. Huse, Bluemel, and Taylor feel much the same way seeing Origami as a way to make "mathematics come alive in the minds of young students…" (1994, p.17). Findings among these authors and many others are very much the same. Origami is seen as a valuable teaching tool; capable of reducing children's anxiety about learning math, increasing their motivation to learn it, and making mathematics more engaging.

Origami and mathematical dispositions

When relating to children's anxiety about learning mathematics, their feelings toward math, and their reactions to mathematics taught, you are considering a child's disposition. NCTM (1989) defines a mathematical disposition as a child's tendency to think positively about mathematics and their willingness to do mathematics. Teachers are seen as key in the assessment and development of dispositions. Teachers are encouraged to listen and observe behaviors of children as they respond to questions, approach new problems, and are presented with difficult tasks.

Origami-mathematics lessons can serve a teacher well as they seek to understand a child's disposition. Origami gives children the chance to see mathematics in another context, approach math concepts in a unique way, and explore their thoughts and ideas about mathematics. Teachers also have a way to see how children approach something new like Origami-mathematics. Are children motivated and curious about it? Do children persevere when the math gets more difficult? Do children seem more confident in their skills when approached in this way? As discussed throughout this article, Origami-mathematics lessons offer children these experiences. Further, teachers can use these lessons as a way to judge children's dispositions. This serves as a valuable tool in evaluating his or her classroom structure, environment, and/or instructional methods.

Folding for the Future

It was the intent of this article to provide readers with a perspective on using Origami in the mathematics classroom. The act of folding an Origami model holds great potential in the classroom. Spatial abilities can be enhanced (Cornelius & Tubis, 2003; Robichaux & Rodrigue, 2003). Terms and concepts can be strengthened (Cipoletti & Wilson, 2004). Further, students can be more engaged and excited about mathematics (Huse et al., 1994; Pearl, 1993). Most importantly, research has been presented that supports these kinds of claims. Origami-mathematics lessons blended a variety of approaches to instruction in a way that students benefited mathematically. While there is much that remains to be done in the area of Origami research, feedback from participants and response teachers share provide evidence enough that it's worth trying. It is the hope of the author that others will continue to delve into Origami, teach Origami-mathematics lessons, and further explore the benefits of this art in the mathematics classroom.

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About the Author:

Norma Boakes is an Assistant Professor of Education at The Richard Stockton College of New Jersey. She was awarded an Ed.D. in *Curriculum, Instruction and Technology Education* from Temple University of Philadelphia, Pennsylvania in 2006. She began her teaching career as a high school mathematics teacher and then later became a faculty member for the School of Education at Stockton. Now in her seventh year as a professor, she actively teaches mathematics content courses for pre-service elementary school teachers, methods of teaching mathematics courses, and a course based on her Origami math research. Her research interests include math attitudes and self-efficacy of teachers of math, alternative methods of mathematics instruction, and Origami in the classroom. Norma can be contacted at Norma.Boakes@stockton.edu

Origami is the ancient Japanese art of paper folding. One uncut square of paper can, in the hands of an origami artist, be folded into a bird, a frog, a sailboat, or a Japanese samurai helmet beetle. Origami can be extraordinarily complicated and intricate. The art of origami has been going through a renaissance over the past 30 years, with new designs being created at ever-increasing levels of complexity. Itâ€™s no coincidence that this rise in origami complexity has emerged at the same time scientists, mathematicians and origami artists themselves have been discovering more and more of the mat... Origami-mathematics lessons simply blend mathematical terminology and discussions within this process. An article found in the online journal Mathitudes details this process and provides an example of what an Origami-mathematics lesson might include (Boakes, 2008). Research with Middle School Students The first research study was conducted in a suburban middle school in southern New Jersey. The purpose of the study was to â€œcompare the spatial visualization abilities and mathematical.Â Boakes, N. (2008). Origami-mathematics lessons: Paper folding as a teaching tool. Mathitudes, 1(1), p.1-9. This study focused on Origamiâ€™s impact as a teaching tool in the middle school mathematics classroom. The effects of Origami instruction on a group of seventh grade mathematics studentsâ€™ (n = 56) spatial visualization skills and level of geometry understanding were investigated using a pre-test/post-test quasi-experimental design.Â One such method is Origami, the art of paper folding. A variety of resource books highlight the geometric nature of the art and the abundance of geometric terminology and mathematical concepts inherent in the folding process (Franco, 1999; Gurkewitz & Arnstein, 1995; Pearl, 1994; Tubis & Mills, 2006).Â The treatment in this study involved the teaching of Origami-mathematics lessons. Origami really does have many educational benefits. Whether you are a student, a teacher, or just a casual surfer, I have tried my best to answer your questions, so please read on.Â These puzzles involve folding a piece of paper so that certain color patterns arise, or so that a shape of a certain area results. But let's continue on with crease patterns Origami, Geometry, and the Kawasaki Theorem.Â There are books and papers published on the subject, presentations given, as well as an international conference called "The International Meeting of Origami Science and Technology". In fact, many of the origami creators and authors here in the US and abroad are mathematicians, physicists, and other scientists.

Origami is the art of paper folding which is often associated with Japanese culture. In modern usage, the word "origami" is used as an inclusive term for all folding practices, regardless of their culture of origin.Â Personally, I have struggled learning and later teaching mathematics over the last 30 years. My quest to search for an answer led me to make new discoveries, one among many was â€˜Origamiâ€™ . So, I went ahead and learnt some basics of origami and was amazed by its power. I have recorded a video, to give you all a sense of how Origami can be used to take mathematics beyond its boring equations and to a realm of intuition, possibilities, imagination and magic. In the video, I will discuss two concepts, one from a lower and one from a higher grade. I have made a video on the same. But what is the limitation of paper folding? What 3D shapes one can make with only folding a single sheet of paper without using scissors and glue?Â Beside Origami he works as a User experience designer at a Tech Company in Tehran. Kiumars Sharifmoghaddam. Tehran, Iran. Kiumars studied Pure Mathematics (B.Sc) and computer science (M.Sc in the field of computational geometry) at University. He started teaching since 2006 when he was just 18 and now he is a Mathematics and Geometry teacher and the head of Mathematics Department at NODET (National Organization for Development of Exceptional Talents) High school in Tehran. In the last 12 years, he has held many creative mathematical and interdisciplinary workshops both in schools and Universities. The discipline of origami or paper folding has received a considerable amount of mathematical study. Fields of interest include a given paper model's flat-foldability (whether the model can be flattened without damaging it), and the use of paper folds to solve up-to cubic mathematical equations. The discipline is often pursued by the use of washi paper. In 1893, Indian civil servant T. Sundara Rao published Geometric Exercises in Paper Folding which used paper folding to demonstrate proofs of... Origami-mathematics lessons simply blend mathematical terminology and discussions within this process. An article found in the online journal Mathitudes details this process and provides an example of what an Origami-mathematics lesson might include (Boakes, 2008). Research with Middle School Students The first research study was conducted in a suburban middle school in southern New Jersey. The purpose of the study was to â€œcompare the spatial visualization abilities and mathematical.Â Origami-mathematics lessons: Paper folding as a teaching tool. Mathitudes, 1(1), p.1-9. Retrieved April 1, 2009 from http://www.coe.fau.edu/mathitudes/20080901bMathitudes_Oct08_revisionFinalVersionforpublica tionOct.%2024,2008.pdf Boakes, N. (2009).