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A NOTE ON CONVEXITY, CONCAVITY, AND GROWTH CONDITIONS IN DISCRETE FRACTIONAL CALCULUS WITH DELTA DIFFERENCE

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Abstract. We demonstrate that some recent results regarding the connection between the convexity of the map $t \mapsto f(t)$ and the sign of $\Delta_a^\nu f(t)$, with $2 < \nu < 3$, can be improved. In particular, by utilizing a recent inequality due to Jia, Erbe, and Peterson, we are able to improve some of the existing results in the literature. As part of this study we illustrate the improvements that our results afford by providing several specific examples of their application.

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