

# Inventory Management with Advanced Warning of Disruptions

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## 1. Introduction

Strategies for managing the risk of supply disruptions include inventory, dual sourcing and contingency (i.e., emergency) sourcing. Tomlin (2005) investigates the optimal mix of these strategies when a supplier faces a constant probability of a disruption. In the constant failure rate case, inventory becomes an increasingly unattractive strategy as the probability of disruption decreases (holding the supplier uptime percentage constant) because inventory needs to be carried for longer durations between disruptions.

In many situations, firms receive some warning that the likelihood of a disruption at a supplier has changed. Such warnings may come in the form of feedback on the progress of labor negotiations, severe weather alerts, or the Department of Homeland Security's terror alert level. We investigate how the presence of such warnings alters the optimal inventory policy, and therefore the optimal disruption-management strategy.

We assume that the firm periodically determines its "threat level," i.e., its estimate of the probability of a supply disruption. The firm can react by adjusting its inventory levels to provide an appropriate level of protection against an impending disruption. Previous models for inventory management under the threat of disruptions have assumed that the firm maintains a constant inventory-level target regardless of the threat level; the advantage of our approach is that inventory levels may remain low when the risk is low and increase only when the risk is heightened.

In this extended abstract, we focus on a special case in which demand is deterministic, the supplier has infinite capacity, and excess inventory may be returned to the supplier at no cost. These assumptions enable closed form solutions to the inventory optimization

problem. In Section 5 of this abstract we discuss relaxations of these assumptions. In addition, we discuss approaches for the more general setting and extensions involving dual- and contingency-sourcing strategies.

## 2. The Model

The firm operates an infinite-horizon, periodic-review inventory system with complete backlogging of unmet demand. The demand is deterministic and constant at  $d$  units per time period. On-hand inventory at the end of a period costs  $h$  per unit, and backorders at the end of a period cost  $p$  per unit. There is a transit lead time of  $L$  periods from the supplier to the firm. We assume throughout that the firm uses the long-run average cost criterion (though many of our results can be adapted to the discounted-cost case).

The firm's supplier is subject to random disruptions, and the failure probability is non-stationary. We assume that the failure probability takes one of  $N$  possible values, and that the firm always knows which probability prevails in a given period. We model this as a discrete-time Markov chain in which there are  $N$  possible "up-states", each corresponding to a different "threat level". During a disruption, no orders can be placed. When the supplier is operational, it has unlimited capacity. We assume that the probability of a disruption ending (and that up-state  $n$  is the first up-state reached after a disruption) depends only the number of periods for which the supplier has been down and the up-state  $n$  in which the disruption occurred. With these assumptions, we can incorporate the supplier's state into the Markov process by adding an infinite number of down-states, indexed by  $(n, i)$ , indicating the number of periods  $i$  (including the current period) for which the supplier has been down and the originating up-state  $n$ . The transition probability from up-state  $m$  to up-state  $n$  is given by  $\lambda_{mn}$ , the probability that a disruption occurs when in up-state  $n$  is given by  $\lambda_{nd}$ , and the probability that a disruption originating in state  $n$  and that has lasted for  $i$  periods ends with the system returning to state  $m$  is  $\lambda_{nim}$ . We denote the steady-state probabilities for the Markov process as  $\pi_u(n)$  and  $\pi_d(n, i)$  for up- and down-states, respectively.

We assume that if the firm's on-hand inventory is greater than its base-stock level, it may return the excess at no charge to the supplier. This situation arises when the firm downgrades its threat level, since (as we will see) the optimal base-stock level increases with the disruption probability. Events occur in period  $t$  in the following sequence:

1. The state of the supplier is observed.

2. Ordering and returns decisions are made.
3. Units produced by the supplier in period  $t - L$  arrive.
4. Demand is filled (if possible) and holding/shortage costs are incurred.
5. The supplier's state transition occurs.

We define the following random variables:

- $x_t$ : on-hand inventory level (positive or negative) at the end of period  $t$ .
- $z_t$ : inventory position (on-hand, on-order, and in-transit) at the end of period  $t$ .

The holding and shortage costs in period  $t$  are given by  $C(x_t) = p[-x_t]^+ + h[x_t]^+$ .

### 3. The Optimal Inventory Policy

**Theorem 1** *A state-dependent base-stock policy is optimal.*

Let  $y_n$  denote the target inventory level at the end of a period in up-state  $n$ , after accounting for the (deterministic) demand. Because returns are allowed at no cost, the inventory position will always equal  $y_n$  at the end of a period in up-state  $n$ . Therefore,  $y_n$  only influences the inventory position in state  $n$  and in the down-states that follow from it; in particular, the base-stock level in state  $n$  does not affect the cost in up-states other than  $n$ . The base-stock optimization problem therefore decomposes into  $n$  separate problems. This is the key fact that makes the free-returns case much more tractable.

**Lemma 2** *The optimal base-stock level in state  $n$  is a multiple of  $d$ .*

Moreover,  $y_n \geq Ld$  for all  $n$ ; otherwise, a shortage cost is unnecessarily incurred in every period. Define  $\hat{y}_n = y_n - Ld$ . Clearly,  $\hat{y}_n$  is also an integer multiple of  $d$ , so we can write  $\hat{y}_n = j_n d$ , where  $j_n = 0, 1, 2, \dots$ . The quantity  $\hat{y}_n$  is analogous to safety stock (protecting against supply uncertainty rather than demand uncertainty), and represents the number of periods of coverage for disruptions beginning in up-state  $n$ . The inventory position will be at least  $Ld$  during the first  $j_n$  periods of a disruption that began in state  $n$  but less than  $Ld$  in later periods of the disruption.

**Theorem 3** *The optimum state-dependent number of periods of coverage is  $j_n^* = F_n^{-1}[\frac{p}{p+h}]$ , where  $F_n[j] = \frac{\pi_u(n) + \sum_{i=1}^j \pi_d(n,i)}{\pi_u(n) + \sum_{i=1}^{\infty} \pi_d(n,i)}$ .*

We therefore obtain a standard newsvendor result in which the probability distribution refers to supply uncertainty rather than demand uncertainty. This result generalizes a single threat-level result from Tomlin (2005). Note that as the probability of disruption increases,  $j_n^*$  increases since the  $\pi_d(n,i)$  increase relative to  $\pi_u(n)$ , thus  $F_n[j]$  decreases and  $F_n^{-1}[j]$  increases.

## 4. The Value of Advanced Information

Consider the special case where the transition probabilities in a disruption are independent of the up-state in which the disruption originated. Define  $\pi_d(i) = \sum_{n=1}^N \pi_d(n,i)$ . If the firm knows the underlying transition probabilities but cannot distinguish between the up-states (i.e. it does not recognize different threat levels), a reasonable policy would be to have a constant base stock level in all up-states, and the so base stock level always provides the same number of periods coverage.

**Theorem 4** *The optimum constant number of periods of coverage is  $j_c^* = F_c^{-1}[\frac{p}{p+h}]$ , where  $F_c[j] = \pi_u(0) + \sum_{i=1}^j \pi_d(i)$ .*

We can then compare the value of advanced information (i.e. the ability to distinguish between threat levels) by evaluating the increase in expected costs if the optimal constant base-stock level is used instead of the optimal state-dependent base-stock levels. A numerical study has found that the difference in costs can be very substantial indicating that implementing an inventory policy that adapts to the threat level can significantly benefit the firm.

## 5. Relaxations

This extended abstract focuses on a special case of the more general model; we assumed deterministic demand, infinite supplier capacity and free returns. To relax these assumptions, we model the more general problems as a Markov decision processes and solve it using a value-iteration method. The infinite capacity assumption made in this extended abstract is a particularly important one, especially if the threat levels have markedly different failure

probabilities and thus markedly different base stock levels. When capacity is finite, the firm may not be able to immediately reach the new base-stock level upon entering a higher threat-level state. The analysis is complicated since the inventory position the firm is able to attain may be different during consecutive periods in the same state. In the finite capacity case, the firm's optimal base-stock levels tend to increase for lower threat levels to ensure that increased inventory targets can be met during heightened threat levels.

## 6. Disruption-Management Strategy

Tomlin (2005) investigates the optimal mix of inventory, dual sourcing and contingency sourcing in an overall disruption-management strategy. We generalize a number of theorems from Tomlin (2005) to the case of an unreliable supplier with multiple threat levels. The presence of multiple threat levels does not alter the set of possible optimal disruption management strategies. Table 3 of Tomlin (2005) still characterizes the optimal strategies. Importantly, however, even for the same supplier uptime percentage, inventory is a more attractive component of the optimal strategy when there are multiple threat levels. The reason is that high levels of inventory are needed only in the high threat-level states. As such, the optimal strategy for a given problem instance is highly dependent on the number of threat levels and the failure-probability in each level. We find that inventory is an important component of the disruption-management strategy over a much larger range of instances in the multiple threat-level case as compared to the single threat-level case.

## References

Tomlin, B.T. 2005. On the value of mitigation and contingency strategies for managing supply-chain disruption risks. Working Paper, Kenan-Flagler Business School, UNC-Chapel Hill, April 2005 version.

Proper inventory management gives you an accurate idea of how many products you sell in a given time, allowing you to forecast sales numbers and optimize how many you need in stock. If you run a tight ship, you'll always have the perfect amount of stock on hand – the ideal balance between meeting demand and minimizing storage costs.

5. Irregular Restocking Schedules. The 5 issues above are all solvable problems. By using Ecomdash's advanced software, each of these issues can disappear through no extra effort on your part: It automatically updates your stock levels in near real time on all your channels, ensuring that your listings are always current, therefore helping to eliminate the possibility of overselling.

Inventory management with advanced warning of disruptions. Working Paper, Kenan-Flagler Business School, University of North Carolina, Chapel Hill, NC, [27] B. Tomlin and Y. Wang. On the value of mix flexibility and dual sourcing in unreliable newsvendor networks. *Manufacturing & Service Operations Management*, 7(1):37-57, [28] J.A. Van Meigham. Risk mitigation in newsvendor networks: Resource diversification, flexibility, sharing and hedging. *Management Science*, 53(8): , [29] C.A. Yano and H.L. Lee. Lot sizing with random yields: A review. *Operations Research*, 43(2): , March-April [30] P.H. Zipkin. *Foundations of Inventory Management*. Advanced Inventory Management (AIM) is a mod designed to automate the inventory of a player. By connecting different devices to a static block network, you are able to manipulate and modify the player's inventory as well as other stats like xp, hunger or potion effects. The mod offers a wide range of different devices and possibilities, reaching from ways to automate your food supply to techniques for auto-healing a player when he has taken damage.