

OUT THERE

In the End, It All Adds Up to $-1/12$

By Dennis Overbye

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This is what happens when you mess with infinity.

You might think that if you simply started adding the natural numbers, 1 plus 2 plus 3 and so on all the way to infinity, you would get a pretty big number. At least I always did.

So it came as a shock to a lot of people when, in a recent video, a pair of physicists purported to prove that this infinite series actually adds up to ...minus $1/12$.

To date some 1.5 million people have viewed this calculation, which plays a key role in modern physics and quantum theory; the answer, as absurd as it sounds, has been verified to many decimal places in lab experiments. After watching the video myself, I checked to make sure I still had my wallet and my watch.

Even the makers of the video, Brady Haran, a journalist, and Ed Copeland and Antonio Padilla, physicists at the University of Nottingham in England, admit there is a certain amount of “hocus-pocus,” or what some mathematicians have called dirty tricks, in their presentation. Which has led to some online grumbling.



But there is broad agreement that a more rigorous approach to the problem gives the same result, as shown by a formula in Joseph Polchinski’s two-volume textbook “String Theory.”

So what’s going on with infinity?

“This calculation is one of the best-kept secrets in math,” said Edward Frenkel, a mathematics professor at the University of California, Berkeley, and author of “Love

and Math: The Heart of Hidden Reality,” (Basic Books, 2013), who was in town recently promoting his book and acting as an ambassador for better math education. “No one on the outside knows about it.”

The great 18th-century mathematician Leonhard Euler, who was born in Switzerland but did most of his work in Berlin and St. Petersburg, Russia, was the first one down this road. Euler wanted to know if you could find an answer to endless sums of numbers like 1 plus $1/2$ plus $1/3$ plus $1/4$ on up to infinity, or the squares of those fractions..

These are all different versions of what has become known as the Riemann zeta function, after Bernhard Riemann, who came along about a century after Euler. The zeta function is one of the more mysterious and celebrated subjects in mathematics, important in the theory of prime numbers, among other things. It was one of the plot threads, for example, in Thomas Pynchon’s 2006 novel, “Against the Day.”

In 1749, Euler used a bag of mathematical tricks to solve the problem of adding the natural numbers from 1 to infinity, a so-called divergent series because the terms

keep growing without limit as you go along. Clearly, if you stop adding anywhere along the way — at a quintillion (1 with 18 zeros after it), say, or a googolplex (10^{100} zeros) — the sum will be enormous. The problem with infinity is that you can't stop. You never get there. It's more of a journey than a destination. As Dr. Padilla says to Mr. Haran at the end of their video, "You have to face infinity, Brady."

The method in the video is essentially the same as Euler's. It involves nothing more complicated than addition and subtraction (although the things being added and subtracted were more infinite series) and a small piece of algebra that my sixth-grade daughter would breeze through.

You are not alone in wondering how this can make sense. The Norwegian mathematician Niels Henrik Abel, whose notion of an Abel sum plays a role here, once wrote, "The divergent series are the invention of the devil, and it is a shame to base on them any demonstration whatsoever."

In modern terms, Dr. Frenkel explained, the gist of the calculations can be interpreted as saying that the infinite

sum has three separate parts: one of which blows up when you go to infinity, one of which goes to zero, and minus $1/12$. The infinite term, he said, just gets thrown away.

And it works. A hundred years later, Riemann used a more advanced and rigorous method, involving imaginary as well as real numbers, to calculate the zeta function and got the same answer: minus $1/12$.

“So Euler guessed it right,” Dr. Frenkel said.

Those of us who are not mathematicians probably wouldn't care so much about infinity except that it crops up again and again in calculations of things, like the energy of the electron, that we know are finite, or in string theory, which physicists would like to hope is finite.

In this case, our current understanding of the very solidity of reality depends on coming up with a consistent way to assign values to infinite sums.

In the process known as regularization, which is a part of many calculations in quantum theory, physicists do something similar to what Euler did, arriving at a real

number that corresponds to the quantity they want to know and an infinite term, which they throw away. The process works so well that theoretical predictions in quantum electrodynamics, the fancy version of the familiar force of electromagnetism, agree with experiments to a precision of one part in a trillion.

Which is remarkable given that infinite quantities have been thrown away, or “swept under the rug,” in the words of the California Institute of Technology physicist Richard Feynman, who helped invent a lot of this stuff but thought it was more than faintly scandalous.

Likewise, it is no surprise that the factor $1/12$ shows up a lot in string theory equations, Dr. Frenkel said. Why it all works is still a mystery.

“Quantum physics needs its own Riemann to come and give a rigorous explanation of these mysteries,” Dr. Frenkel said.

To him and others, this is just another example of what the eminent physicist Eugene Wigner called the “unreasonable effectiveness of mathematics.” Why should such woolly and abstract concepts as zeta functions or imaginary numbers, the products of a chess

game in our minds, have such relevance in describing the world?

Riemann's explorations of the geometry of curved spaces in 1854 laid the foundation for Einstein's theory of gravity, general relativity, half a century later.

There were mathematicians and philosophers who were ready to jump out the window later in the 1800s when Georg Cantor, a Russian-born mathematician, set out to classify the kinds of infinity. In a speech in 1908, the French mathematician Henri Poincaré compared "Cantorism," as he called it, to a disease.

Mathematicians today agree that there is an infinite number of natural numbers (1, 2, 3 and so on) on the bottom rung of infinity. Above that, however, is another rung of so-called real numbers, which is bigger in the sense that there is an uncountable number of them for every natural number. And so it goes.

Cosmologists do not know if the universe is physically infinite in either space or time, or what it means if it is or isn't. Or if these are even sensible questions. They don't know whether someday they will find that higher orders of infinity are unreasonably effective in understanding

existence, whatever that is.

Here is where we sprain our imaginations, and perhaps check to see that we still have our wallets.

If you need to be able to do this for many paragraphs, set the tab stop in the style definition for the type of paragraph. For example, if you wanted the filled dots for a paragraph you have tagged with the Body Text 2 style, add the tab stop to its definition. Originally Answered: How can you add automatically continuous dot in the end of sentences until the right margin of Microsoft word? I'm not sure of the Quora rules on just posting an URL as an answer to a question, but here it goes: Add dot leaders to a table of contents. How can I set it up so that changes or additions to one Microsoft word document will result in automatic changes to another related word document? About · Careers · Privacy · Terms · Contact · Languages · Your Ad Choices · © Quora Inc. 2020. [Chester Bennington:] In the end [Mike Shinoda:] You kept everything inside and even though I tried, it all fell apart What it meant to me will eventually be a memory of a time when I tried so hard. [Chester Bennington:] I tried so hard And got so far But in the end It doesn't even matter I had to fall To lose it all But in the end It doesn't even matter. I've put my trust in you Pushed as far as I can go For all this There's only one thing you should know. I've put my trust in you Pushed as far as I can go For all this There's only one thing you should know. I tried so hard And got so far But Sign up or log in to customize your list. more stack exchange communities. company blog. Join Stack Overflow to learn, share knowledge, and build your career. Sign up with email Sign up. 12.1k55 gold badges2525 silver badges4747 bronze badges. 4. 4. The empty line in the end of file appears so that standard reading from the input stream will know when to terminate the read, usually returns EOF to indicate that you have reached the end. The majority of languages can handle the EOF marker. It is there for that reason from the old days, under DOS, the EOF marker was F6 key or Ctrl-Z, for *nix systems, it was Ctrl-D. Most, if not all, will actually read right up to the EOF marker so that the runtime library's function of reading from input will know when to stop reading any further. Sign up or log in to customize your list. more stack exchange communities. company blog. I have 23,000 lines, and I need to add the number 1 at the end of every line. How can I do this? Right now Then again, xkcd.com/1171 "David Z Jul 21 '15 at 12:52. 1. This isn't a very good answer, first you would need to find the longest line, then you would need to press down till the last line, while testing this method I got bored after a couple dozen lines, I could not sit there for 20k+ lines "Aequitas Jul 23 '15 at 5:35. | Show 2 more comments. 2. Extended Mode. I wanted to add a slightly different approach that's not mentioned in other answers: Extended Mode. In the end is often used to signify that something has occurred after consideration of various options and a period of uncertainty. So, We looked at several pianos. They all had attributes we liked but in the end it was the small size of the upright that we went for. At the end is usually used simply to specify a moment in time. So, At the end of the game players add up how much money they each have and the winner is the person with the least (!) Share. Improve this answer.

1. To bring to a conclusion: Let's end this discussion. 2. To form the last or concluding part of: the song that ended the performance. See Synonyms at complete. 3. To destroy: ended our hopes. v.intr. 1. To come to a finish; cease: The rain ended. 2. To arrive at a place, situation, or condition as a result of a course of action. Often used with up: He ended up as an adviser to the president. The painting ended up being sold for a million dollars. 3. To die. Idioms: at the end of (one's) rope/tether. Out of energy or patience; exhausted or exasperated. at the end of the day. When everything is considered; in the final analysis. end it all. To commit suicide. in the end. Eventually; ultimately: All will turn out well in the end. no end. A great deal: She had no end of stories to tell. "19 July is not only the end of the line but the start of an exciting new journey for our country." Health Secretary Sajid Javid has updated MPs on the planned ending of England's lockdown. He confirms that the final stage in ending the lockdown remains 19 July. Javid says: "My task is to help return the economic and cultural life of this country". He says that by 19 July two thirds of adults will have had both coronavirus jabs. He said that while cases were rising, the number of deaths "remains mercifully low". Many firms, especially in the night-time hospitality sector, had budgeted to be able to trade from 21 June, it says. A government spokeswoman says "substantial" support, including the furlough scheme and grants, will remain in place. Read more. The End of All Things to Come is the second studio album by American heavy metal band Mudvayne. Released on November 19, 2002, the album expanded upon the sound of the band's first album, L.D. 50, with a more versatile range of sounds, dynamic, moods and vocalization. The band wrote the album's songs in less than a month, drawing inspiration from their self-imposed isolation during the songwriting process, and crafted a more mature sound which drew from jazz and progressive rock influences, as well as... Sign up or log in to customize your list. more stack exchange communities. company blog. Super User is a question and answer site for computer enthusiasts and power users. It only takes a minute to sign up. Sign up to join this community. Anybody can ask a question. Anybody can answer. I have 23,000 lines, and I need to add the number 1 at the end of every line. How can I do this? Right now Then again, xkcd.com/1171 " David Z Jul 21 '15 at 12:52. 1. This isn't a very good answer, first you would need to find the longest line, then you would need to press down till the last line, while testing this method I got bored after a couple dozen lines, I could not sit there for 20k+ lines " Aequitas Jul 23 '15 at 5:35. | Show 2 more comments. 2. Extended Mode.