EXPERIMENTAL STUDY ON THE HIGHER ORDER NONLINEAR RESONANCE AT TAIWAN LIGHT SOURCE

T. S. Ueng, Jenny Chen, K. T. Hsu, K. H. Hu, W. K. Lau, SRRC, Hsinchu, Taiwan

Abstract

The higher order nonlinear resonances of transverse betatron oscillation were studied experimentally at the SRRC's storage ring. The turn-by-turn method and phase space map were used to analyze the data. The instantaneous tune was extracted to study the tune variation from the different betatron amplitude. The linear coupling due to the difference resonance at around the 6th order nonlinear resonance has also been investigated.

1 INTRODUCTION

The nonlinear beam dynamics study is continuing at SRRC's Taiwan Light Source (TLS)[1,2], a combined function TBA lattice type storage ring. The effects of higher order multipoles errors driving the higher order nonlinear resonances are under investigating. The general resonance condition is $m\nu_x \pm n\nu_y = p$, where ν_x and ν_y are horizontal and vertical tunes respectively, and m, n, p are integers. If p is a multiple of the accelerator's superperiodicity, the resonance excitation will be very sensitive to the systematic errors in the lattice elements. The TLS storage ring has the superperiodicity of 6. For the one-dimensional non-linear resonance, $m\nu_x = p$. The 5^{th} order nonlinear resonance with m = 5 and $\nu_x = 7.2$ gives p = 36. It is a multiple of 6 of the superperiod of the TLS storage ring. Thus, a study on the 5^{th} order nonlinear resonance is interesting.

In order to extend the nonlinear resonance study to 6^{th} order, the horizontal tune was adjusted to $\nu_x \sim 43/6$. A linear coupling from the difference resonance appeared at around $\nu_x \sim 7.166$ and $\nu_y \sim 4.166$. The effect of the coupling complicated the investigation of the 6^{th} order resonance. A preliminary study of this coupling phenomenon was also reported.

The extraction of instantaneous tune is interesting in some applications of the beam physics. With the numerical analysis of fundamental frequency (NAFF) method[3] the instantaneous tune can be extracted with a small number of turns of beam positions. It is used in this report to study the decoherence effect and also to obtain some nonlinear resonance parameters.

2 EXPERIMENTAL PROCEDURE

The data acquisition system used for the turn-by-turn beam position measurement was the same one used in the previous studies with performance enhanced. The vertical beam position could be acquired more accurately. One of the injection kickers was used to kick the electron beam horizontally to produce the coherent betatron motion. The beam position signals were measured by beam position monitors, then, passed through a hybrid junction and Bergoz's Log-Ratio beam position monitor electronics (LR-BPM) to the VME based transient digitizers. The online data acquisition can be initiated with a workstation installed with GUI based software. The raw data and the result could be displayed on line and the saved beam positions would be used for further analysis. During the experiment it was found that the multi-bunch beam was adequate for the experiment.

3 RESULT AND DISCUSSION

3.1 Instantaneous tune

The instantaneous tune can be extracted with the NAFF method proposed by Laskar. The instantaneous tune $\nu_{m,N}$ is the tune with N consequent turns starting at turn m by maximizing the absolute value of $I(\nu_{m,N})$.

$$I(\nu_{m,N}) = \sum_{n=m}^{m+N-1} f_n \cdot \exp(-i2\pi \cdot \nu_{m,N} n)\chi_{m-n} \qquad (1)$$

$$\chi_{m-n} = \sin(\pi \cdot (m-n)/N)$$

f_n is the turn-by-turn beam position at n-th turn. $\nu_{m,N}$ is the fundamental frequency, or the tune. x_{m-n} is the weight function to improve the precision of extracted tune.

The decoherence due to the tune spread results in the amplitudes of the centroid of the beam positions measured by beam position monitor decay rapidly as the kicked amplitude of the electron beam increased. In order to measure the instantaneous tune at different amplitude during the rapid decaying of the beam position amplitude, only a small number of turns can be used. The traditional plain fast fourier transform (fft) methods using a large number of beam positions is not suitable. Thus, the NAFF method provides an alternative solution to obtain the amplitude dependent tune shift with rapidly decaying amplitude. In Fig. 1, the instantaneous tune obtained was plotted vs. the square of the instantaneous amplitude. The instantaneous amplitude of measured beam position was obtained with the amplitude of the complex Hilbert transform. In the figure, the circles represent the tune obtained with the plain fft method using the whole set of the beam positions. It shows that the TLS has a negative detune value about -1.11×10^{-3} (mm mrad)$^{-1}$ at the condition used for this measurement. The straight line is the linear fit to these data points. The other curves represent the instantaneous tunes obtained with 3 arbitrarily selected data sets of beam positions using N=256. While at the maximum beam positions right after the kicks the tunes obtained with both methods give about the same results. But the instantaneous tunes obtained with NAFF method show that the tunes do not decrease with the square of the amplitude linearly. The tune have small slope at higher amplitude, but larger at smaller amplitude. At the smaller amplitude the beam positions included for the calculation might contain the already damped beam positions. This could make the calculated instantaneous tune incorrect. A more detailed study shows that the number of points, N, selected will make the instantaneous tune curve different slightly at the lower amplitude. A selection of larger N will give a little higher tune, i.e. close to the fitted curve. Thus, the number N used for the calculation is critical. At the higher amplitude the calculated instantaneous tune is lower than that using plain fft method. This might be due to the fact the beam position measured under the decoherence is the centroid of the particles in the beam. The actual beam positions of the particles might be still larger. Thus the instantaneous tune still shows larger and close to the original just kicked beam. There might still have other effects, a detailed study is needed in order to understand it.

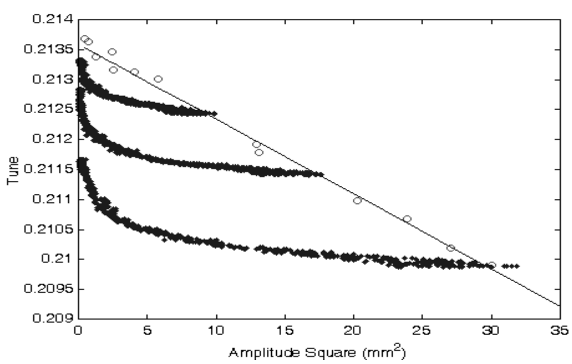

Figure 1: Comparison between the instantaneous tune vs. the square of the amplitude and the tune measured with plain fft method using the whole set of beam position data.

3.2 5th order Nonlinear resonance

The 5th order nonlinear resonance data was taken with the chromaticity set to about zero. The nonlinear detuning parameter, α_{xx}, at this condition was about -1.11×10^{-3} (mm mrad)$^{-1}$. Because of the negative nonlinear detuning effect of the TLS, the data taking was started at ν_x slightly higher than 7.2 and the beam was kicked from lower kicking strength until it could be kicked out of the ring. When all of the required conditions were met, the 5th order nonlinear resonance could be obtained. In Fig. 2, one set of the turn-by-turn beam positions of the one-dimensional 5th order resonance and its fft spectrum are shown. Fig.3 shows its phase space map in the normal coordinates, and its corresponding map in action-angle variables (J_x and φ). This spectrum contains 13 msec of data points. It shows that the resonance can stay at almost the same amplitude for a long period of time. The island tune ν_{island} can be seen in the side band of the main peak, ν_x~7.2, of the fft spectrum. ν_{island} is estimated about 0.0018, which corresponds 555 orbital turns.

The Hamiltonian for the 5th order nonlinear resonance can be written as

$$H = \delta \cdot J_x + \frac{1}{2}\alpha_{xx} \cdot J_x^2 + G_{5,0,l} J_x^{5/2} \cos 5\Phi \qquad (2)$$

where $\delta = \nu_x - \ell/5$ is the resonance proximity parameter; $\Phi = \phi_x - \ell/5 \cdot \theta + \xi/5$; J_x and ϕ_x are conjugate action-angle variables. G and ξ are the resonance strength and phase; θ is the orbital angle. In the present preliminary study, a simplified Hamiltonian is used. Near the betatron resonance at $m\nu_x(J_r) = p$, J_r is the stable fixed point. The Hamiltonian can be approximated by [4,5]

$$H = \frac{\alpha}{2}(J - J_r)^2 + g(J_r)\cos(m\phi - \chi) + \cdots \qquad (3)$$

One set of our data of the island ellipses in the action-angle variables phase space map was fitted with the Hamiltonian of Eqn.(3). Its result is shown in Fig. 4. The resonance strength was estimated about 9×10^{-4} (mm mrad)$^{-1}$. A more through study using the Hamiltonian in Eqn.(2) is underway for understanding the detailed mechanism of the 5th order nonlinear resonance.

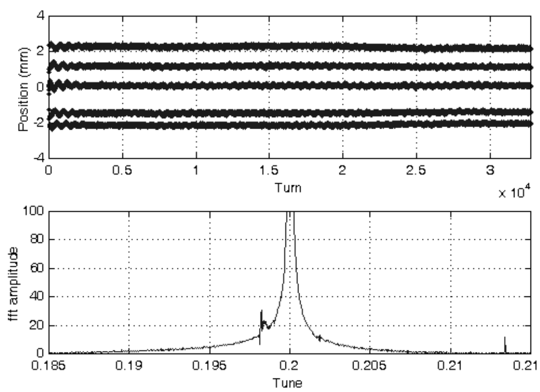

Figure 2: A 32000 turn beam positions vs. turn number (above) and its corresponding fft spectrum in the ν_x range from 0.185 to 0.215 (below).

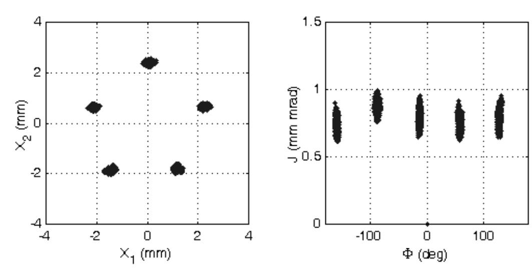

Figure 3: The corresponding phase space map of data in Fig. 2, in normal coordinates (left), in action- angle variables (right).

3.3 Linear coupling around ν_x=43/6

During the exploration of the 6th order nonlinear resonance, the quadrupole strength was adjusted to lower the horizontal tune from that used in the present user shift (ν_x ~ 7.22, ν_y ~ 1.40) to ν_x ~7.167. At the same time the vertical tune was also increased following the relation between the horizontal betatron function and vertical betatron function at the adjusted quadrupole. At around fractional tune of $\nu_x \sim 1/6$, ν_y could also have fractional tune very close to 1/6. With the skew quadrupoles' strength set at some value, it was found that the horizontal betatron motion could couple with the vertical betatron motion. Its turn-by-turn beam position vs. turn number plot and phase space plot could be similar to those of a 6th order nonlinear resonance, Fig 5. After studied carefully, the data shown in Fig. 5 was identified as the result of the linear coupling. From the beating period of beam positions vs. turn number one could find that it corresponds to tune of 0.0172. It is about the tune difference of the horizontal and vertical tune, ~0.0172, obtained from the fft spectrum of the beam positions. With the instantaneous beam positions obtained with the Hilbert transform, one was able to obtain the ratio of the minimum of the beam position to the maximum, G = 0.45. The distance of the tunes from the coupling resonance, $|\Delta|$, and the coupling coefficient, $|C|$, can be obtained with

$$|\Delta| = \frac{G}{f_{rev} \cdot T}$$

$$|C| = \frac{1}{f_{rev} \cdot T}\sqrt{(1-G^2)}$$

where f_{rev} is the revolution frequency of the accelerator, it is 2.5 MHz at TLS, and T is the beating period. For the data shown in the figure, $|\Delta| = 0.008$, and $|C| = 0.015$. It indicated a strong coupling effect occurred when the data was taken. If the 6th order nonlinear resonance is to be studied, the coupling effect must be suppressed at fractional tune of ν_x at around 1/6. So, it will not complicate the analysis of the 6th order nonlinear resonance.

4 CONCLUSION

A preliminary study on the 5th order nonlinear resonance at $\nu_x = 7.2$ has been done. Its dynamics of resonance islands was investigated. During the study of the nonlinear resonance, the NAFF instantaneous tune extraction method was also used and studied. For the exploration of 6th order nonlinear resonance, the difference coupling was observed and investigated in order to simplify the studied of the 6th order nonlinear resonance in the future. At present, the turn-by-turn beam position data acquisition system has been improved with better resolution, and the vertical beam positions can be measured along with the horizontal beam positions easily. The coupling effect and 2-dimensional resonance will be studied in the near future.

5 ACKNOWLEDGEMENT

The authors are very appreciated the assistance in operating the accelerator from the members of SRRC's Operation Group during experiments. With thanks to Prof. S. Y. Lee of IUCF for very helpful discussion.

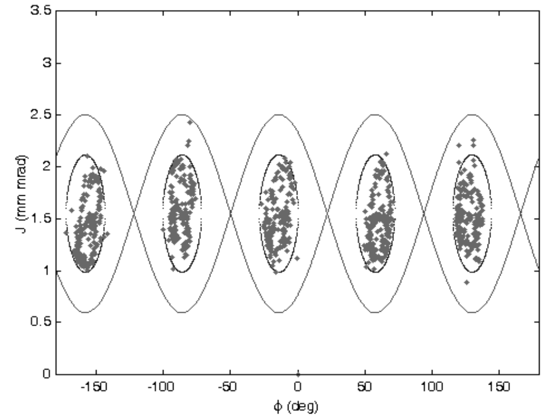

Fig. 4: The stable ellipses around island fixed points in the action angle variables were fitted by a simplified Hamiltonian of Eq. (3).

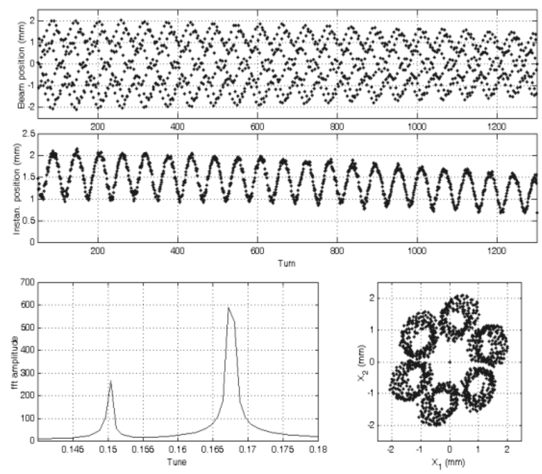

Figure 5: The difference coupling appears as the tune of ν_x approaching 43/6.

6 REFERENCES

[1] T. S. Ueng et al., "Experimental Study on the 5th Order Nonlinear Resonance at Taiwan Light Source," PAC 2001, Chicago, June 2001.

[2] C. C. Kuo et al., "Nonlinear Beam Dynamics Experiments at the SRRC," PAC 2001, Chicago, June 2001.

[3] A. Terebilo, et al., "Experimental Nonlinear Beam Dynamics Studies at SPEAR," PAC 1997, Vancouver, June 1997.

[4] S.Y. Lee et al., "Experimental Determination of a Nonlinear Hamiltonian in a Synchrotron," Phys. Rev. Lett.. 67, 3768, 1991.

[5] S.Y. Lee, "Review of Nonlinear Beam Dynamics Experiments," Inst. Phys. Conf. Ser. No 131, 249, 1993.

Higher-order components in the magnetic field in a ring introduce nonlinear terms into the Hamiltonian and generate nonlinear resonances. This can lead to complicated motion for particles with large amplitudes of betatron oscillations. We derive the resonant structure in the phase space due to a sextupole magnet when the fractional part of the tune is close to \(\pm \frac{1}{3}\). For a Hamiltonian system with many resonances, they can interact with each other and lead to stochastic orbits in phase space. To understand this effect, we study a model called the standard map, that illustrates qualitative features of what can occur in a Hamiltonian system with many resonances. In general, nonlinearities of any order can be consid-ered; however, for simplicity, we first examine only a cu-bic nonlinearity where the index of refraction n is ex-pressed in terms of nonlinear indexes n2(esu) or. y (m2. / W) through. Fig. 1 . The Z-scan experimental apparatus in which the ratio 0 2 / 0 1 IS recorded as a function of the sample position L. beam divergence, leading to beam broadening at the ap-erture, and thus a decrease in transmittance. This suggests that there is a null as the sample crosses the focal plane.Â The on-axis electric field at the aperture plane can be obtained by let-ting r = 0 in (9). Furthermore, in the limit of small non-. linear phase change (. I. Description of linear and second-order-nonlinear properties of the used materials; Analysis of regularities and attempts to predict characteristics of new compound; Modelling of light propagation in nonlinear inhomogeneous media with specially fabri-cated structure, resulting in some unusual properties. Based on this the present work has the following structure: In the first chapter, methods of deposition of dye-doped polymer films are discussed. Vari-ous techniques are discussed and compared â€" spin-coating, dip-coating, drop-casting, and vacuum deposition. Although all of these techniques are... In this experimental case study, identification of the nonlinearity in the system and model. 3. updating of the linear part are both performed. Accuracy of the proposed method is demonstrated on experimental measurements. Chapter 7 discusses the results obtained from previous chapters. The conclusion of thesis is given in this chapter.Â Newtonâ€™s method is one of the popular root-finding numerical solution techniques based on the first order Taylor series expansion. The iterative formula for Newtonâ€™s. method is given as follows.