Coherent Motions in the Fermi-Pasta-Ulam Model

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Lecture 1: The Fermi-Pasta-Ulam model; introduction and KdV approximation

Outline:

1. Models of coupled oscillators
2. Two main classes
   (a) Models with on-site potentials; e.g. Frenkel-Kontorova or discrete NLS
   (b) Models with only nearest neighbor coupling; e.g. FPU type models
3. Focus on:
   (a) localized oscillations
   (b) traveling waves
4. Numerical experiments of Kruskal and Zabusky and the discovery of solitons.
5. The formal approximation of the FPU model by the KdV equation
6. A general method of justifying approximations by modulation equations
7. Details of the approximation proof in the case of the FPU model.

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Lecture 2: Discrete Breathers

Outline:

1. Periodic solutions in nonlinear equations and discrete breaters
2. Periodic solutions in the linearized FPU model
3. Approaches for constructing discrete breathers in the FPU model:
   (a) Variational Methods
   (b) Methods based on the Implicit Function Theorem
      i. Continue solutions from the "anti-integrable" limit, rather than the linear problem
   (c) Center-manifold methods
      i. Toda's "dual formulation" of the FPU equations.
      ii. A dynamical systems on the space of loops
      iii. Infinite dimensional map on the space of Fourier coefficients
      iv. The equation on the center-manifold.
      v. Possible multi-scale expansions for breathers.

Lecture 3: Traveling Waves in the FPU model

Outline:

1. The Toda Lattice
   (a) Explicit form of the one and two-soliton solutions.
2. Traveling waves in more general FPU models
   (a) Variational Approach
   (b) Center-manifold Approach
   (c) "Renormalization" approach
3. The Friesecke-Pego renormalization approach
   (a) Relation of the traveling wave profile to the KdV equation
   (b) Stability of traveling waves
      i. What kind of stability can one expect?
   (c) Modulation equations
   (d) Linear stability implies nonlinear stability

Lecture 4: Counterpropagating two-solitons in the FPU model

Outline:

1. Linear Estimates:
   (a) Bäcklund Transformations for the Toda model.
   (b) A perturbation argument for general FPU solitons in the KdV limit.
2. The two-soliton problem
   (a) Localizing the perturbation
   (b) Controlling the interaction
   (c) Differences with stability problems
3. Statement of results
4. Sketch of proof

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The Fermiâ€“Pastaâ€“Ulam problem and its underlying integrable dynamics. G. Benettin. âˆ—.Â This paper is devoted to the Fermiâ€“Pastaâ€“Ulam (FPU) model [1], more precisely to the soâ€“called â€œÎ±+Î²â€ model with Hamiltonian. 1 H(p, q) =. 2.Â This thought however prompted us to study the Toda constants of motion in the FPU dynamics, as will be explained in the next section. â€“ High modes are more sensitive than low modes to the nonâ€“integrability of the FPU model. Their decorrelation, larger in FPU than in Toda, shows, before low modes, that the FPU model, on a larger time scale, is chaotic and ultimately obeys statistical mechanics; low modes are lazier. The Fermiâ€”Pastaâ€”Ulam (FPU) paradox 4.2. The Kolmogorovâ€”Arnolâ€™dâ€”Moser (KAM) theorem. and nonlinear resonances.Â possible modes, at least in the vicinity of a multidimensional surface in phase space that separates reagents from products. This latter construct is called the â€œdividing surfaceâ€ (for reviews, see Hase [1976], Truhlar et al. MATLAB Fermi-Pasta-Ulam lattice model can't force periodic boundary condition. Ask Question. Asked 9 years, 2 months ago.Â I am building MATLAB code to implement a symplectic integrator using a second order split-step method for the Fermi-Pasta-Ulam problem. The Hamiltonian is in the form: H = sum from i=1 to N {0.5*p_i^2 + 0.5*(q_[i+1] - q_i)^2+(a/3)*(q_[i+1] - q_i)^3 + (b/4)*(q_[i+1] - q_i)^4}. I assume a boundary condition: q_[N+1]=q_1 and p_[N+1]=p_1. So I have my oscillators on a string in a circle. The trouble I have is I can't seem to force the periodic boundary condition! Here is my main program and the grad of Hamiltonian in a separate file, I think I have x0 wrong but not sure how to improve on it! The sine wave gets all distorted on the ends. Hamiltonian.m