
Principles of Optimal Design

Modeling and Computation

THIRD EDITION

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Principles of Optimal Design

Modeling and Computation, third edition

Design optimization is a standard concept in engineering design, and in other disciplines which utilize mathematical decision-making methods. This textbook focuses on the close relationship between a design problem's mathematical model and the solution-driven methods which optimize it. Along with extensive material on modeling problems, this book also features useful techniques for checking whether a model is suitable for computational treatment. Throughout, key concepts are discussed in the context of why and when a particular algorithm may be successful, and a large number of examples demonstrate the theory or method right after it is presented. This book also contains step-by-step instructions for executing a design optimization project – from building the problem statement to interpreting the computer results. All chapters contain exercises from which instructors can easily build quizzes, and a chapter on “principles and practice” offers the reader tips and guidance based on the authors' vast research and instruction experience.

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Principles of Optimal Design puts the concept of optimal design on a rigorous foundation and demonstrates the intimate relationship between the mathematical model that describes a design and the solution methods that optimize it. Since the first edition was published, computers have become ever more powerful, design engineers are tackling more complex systems, and the term "optimization" is now routinely used to denote a design process with increased speed and quality. This second edition takes account of these developments and brings the original text thoroughly up to date. The book now includes a discussion of trust region and convex approximation algorithms. A new chapter focuses on how to construct optimal design models. Principles of Optimal Design puts the concept of optimal design on a rigorous foundation and demonstrates the intimate relationship between the mathematical model that describes a design and the solution methods that optimize it. Since the first edition was published, computers have become ever more powerful, design engineers are tackling more complex systems, and the term optimization is now routinely used to denote a design process with increased speed and quality. This second edition takes account of these developments and brings the original text thoroughly up to date. The book now includes a discussion of trust region and convex approximation algorithms. A new chapter focuses on how to construct optimal design models.

2. optimal design principle. Both the structural organization and perfect operation of biological systems, are the result of the specific objective laws and principles, guiding the structuring and functioning of living matter. One of the universal principles, acting upon living matter, was called [8] Optimal Design Principle. (ODP). As any principle, ODP cannot be demonstrated or proved directly, but it is considered to be valid.

1. POPESCU, A. I., Biological System and the Principle of Optimal Design, Acta Biotheoretica, 46, pp. 299-310, 1998/1999. 2. BASTIAN, J., Electrosensory Organisms, Physics Today, February: pp. 30-37, 1994. 3. SELIGER, H. H., LALL, A., LLOYD, J. E., BIGGLEY, W. H., The Colours of Firefly Bioluminescence. Optimal design is usually considered as the design process that seeks the "best" possible solution(s) for a mechanical structure, device, or system, satisfying the requirements and leading to the "best" performance, through optimization techniques. It also refers to the design points that best satisfy objective(s) which are in contrast to non-optimal design. George Dieter has given a formal definition of design as follows: "Design establishes and defines solutions to pertinent structures for problems not solved before, or new solutions to problems which have previously been solved in a different way" (Dieter and Schmidt 2009). Optimal indicates a searching and decision-making process that is required to determine the best possible design alternatives.