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Høyrup, Jens

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THE STATUS OF THEORY AND PRACTICE IN ANCIENT, ISLAMIC AND MEDIEVAL LATIN CONTEXTS

Jens Høyrup

As all contributors to the present volume know, Laura Toti Rigatelli has worked in in various fields within the history of mathematics. However, my own working catalogue, my bookshelves and my archive indicate clearly that what has affected me most is her work on the late medieval and Renaissance Italian practitioners' *abbaco* tradition. My homage to her will therefore deal with the status of practitioners' knowledge.

It is to a large extent to the credit of Laura Toti Rigatelli (together with Raffaella Franci and Gino Arrighi) that interest in practical mathematics is not uncommon within Italian historiography of mathematics. In the perspective of the global community of historians of mathematics, however, things appear to look different; here, work on theoretical knowledge by far overshadows that on the knowledge of pratitioners. Moreover, since the knowledge of specialists' crafts is generally considered of scarce interest, it is largely overlooked that it was largely independent of what we are accustomed to see as contemporary "scientific knowledge", at least during the pre-Modern epoch and to some extent even later, *but* that it is none the less not to be understood as mere and general "folk knowledge".

As Laura Toti Rigatelli I have mainly approached the relationship between the two types and organizations of knowledge from the point of view of mathematics, which is the field I know best [Høyrup 1990; 1997a]. This may be claimed to have distorted my view, since a corpus of knowledge as coherent as that of "scientific" (Euclidean, Archimedean, etc.) mathematics is not found in other fields in the pre-Modern world. Scientific mathematics can be argued to have taken over material and inspiration from the practical traditions and to have digested it in such a way that wholly new structures emerged; In other fields, "philosophers" made common-sense use within their own framework of every-day observations or of knowledge deriving from the practical traditions – but they did not transform this borrowed knowledge radically.

This may to a large extent be true.[1] I shall argue, however, that it is not always true; I will also claim that the case of mathematics reveals tendencies which are everywhere present by have not crystallized as clearly in other domains as in the case of mathematics.

In [1987] I tried to launch the concept of an "Islamic miracle", a counterpart of its better-known "Greek" namesake. While the "Greek miracle" consists, briefly spoken, in the discovery of autonomous knowledge as a possibility, its "Islamic" counterpart is (ideally speaking) the discovery that no practice is too mean to serve as a starting point for theoretical reflection, and no theory too lofty to be applied in practice – a discovery which in the European and particularly

[1] It can even be seen to be true when mathematically less competent ancient philosophers like Plutarch and Iamblichos (mentioned as embodiments of the whole neopythagorean current) appropriated part of practical arithmetical knowledge – cf. [Høyrup 2000].

the British tradition is mostly referred to Francis Bacon.[2]

In what follows I shall try to convey some of the insights into the diverse (not to say divergent) approaches to knowledge in the Ancient, Medieval Islamic and Medieval Latin world which I believe to have gained from this work – presenting first some examples of contrasting ways in which different cultures have approached same subject-matter, and drawing afterwards some general conclusions.

Examples in contrast

The first example comes from basic geometrical construction. It is the question, *how to draw a straight line*.

In a way, this is the topic of Postulate 1 of the *Elements* [trans. Heath 1926: I, 154], according to which it is possible "to draw a straight line from any point to any point". The Greek verbal form, however, is a perfect infinitive ἀγαγεῖν. As pointed out by Marinus Taisbak [1992: 144], this linguistic peculiarity is all-pervasive in the *Elements* (and the *Data*): lines are always drawn, points taken, and perpendiculars dropped in the perfect imperative passive; in Taisbak's words, actual drawing is taken care of by an ever-present "Helping Hand". The actual performance of the act does not concern the geometer.

In contrast, Chapter I of Abū'l-Wafā'ʾs *Book on What is Necessary for Artisans in Geometrical Construction* starts as follows [Russian trans. Krasnova 1966: 56*f*]:

> Know that the correctness or incorrectness of a construction depends on three things: The ruler, the compass, and the try square.
>
> The most important of these is the ruler. That is, a straight line without bend. As Archimedes said, it is the shortest line that connects two points, for example the points *A* and *B* and reversely [reference to a drawing in the treatise]. If from one of them to the other several lines are drawn, for example the line *ACB*, *ADB* and *AEB*, then the shortest of these ist the straight line, namely the line *ACB* [straight in the drawing]. Therefore, if we have a ruler and its edge lays along a straight line, then the ruler in question is correct. Such rulers are used for short straights and lines. But if the straight or the line is long, then ropes are used.
>
> A ruler is corrected by grinding. If the one which we need to correct is very long, then it is split into several ropes which are then corrected by grinding. If we want to correct a ruler we must first of all smooth its edge if it is made of any hard material, be it iron or copper or anything similar; or level it with an axe, if it is made of wood, and then correct it by grinding. If we have finished the correction of the ruler and want to find out afterwards whether we have performed the correction properly, then we put it in an even place and mark the position of its straight edge, and then interchange the places of its ends and again mark the position of its straight edge. If these two straights coincide, then the ruler is correct, if however they do not coincide, then we know in which place there is a bend, since it is in the place where the two straights deviate and do not coincide.
>
> Most artisans control the correctness of a ruler visually. When we look from one end to the other, then it is obvious in which place it deviates upwards, and in which downwards. This happens because of the straightness of the light ray.

One might of course be tempted to explain (that is, explain away) the difference in perspective from the diversity of the audiences for which the two texts were written. In order to test this

[2] Knowing well the writings of Renaissance artists, experimentalists and engineers, Paolo Rossi [1970: 101 and *passim*] is quite aware that these made similar claims long before Bacon. Even in his reading of the record, however, Bacon is the figure who symbolizes the acceptance of the view *within philosophy*.

explanation we may look at the way theoretical insights appear in Hero's *Metrica*.

Metrica deals with mensurational computation and not with construction, it is true, and therefore does not discuss the drawing of lines; but its way to integrate theoretical results into a practical handbook remains informative regarding the way it understands that integration of theory and practice which is the project.

I shall not go through the argument in detail, as I have done that elsewhere [Høyrup 1997b], but only repeat the conclusions which can be drawn on the basis of the way "Hero's formula" and the Archimedean approximation to a flat circular segment occur in the text:

Hero uses a pre-existing practical handbook rooted in the practitioners', not the Euclidean tradition, and within each main chapter he follows its order section for section;[3] if the contents of a specific section seems unsatisfactory, however, he replaces it with something borrowed from theoretical geometry (e.g., the extended Pythagorean theorem instead of the computation of the projections of triangular sides by means of quasi-algebraic methods); he inserts extra chapters when he thinks that some formula not covered in the original should be included, and provides them with the necessary lemmata; but he does not smooth the text and does not care about the coherence between the "theoretically based" parts of the text and the rest; thus he will argue within of one of the "theoretically based" insertions that a particular formula should not be used when the base of a circular segment is more than thrice the arrow,– only to use the very same formula in the next "practitioner-based" chapter in the case where the ratio is 4.

This work, we should remember, is the best surviving attempt from Greek antiquity to make something of the same kind as Abū'l-Wafāᵓ's work; when at all using material of "scientific" extraction, the pseudo-Heronian conglomerates *Geometrica* and *De mensuris* are even less oriented toward integration of the theoretical and the practical perspective. Even when really trying to accomplish something similar to what Abū'l-Wafāᵓ was eventually to achieve, no mathematical writer in Antiquity was really able to do so. As soon as it was present, theoretical mathematics impressed upon the writer a concern for metatheoretical pureness that prevented genuine integration. Once more, Hero may serve as an example. When transferring "Hero's formula" from the practically oriented treatise on the *Dioptra* to the *Metrica*,[4] two characteristic changes are made. Firstly, a lemma is added according to which the "side" of a rectangle[5] whose sides are the squares on lines AB and BΓ is equal to the rectangle contained by AB and BΓ; this theorem, used as practical and self-evident knowledge already in the Old Babylonian problem BM 13901 no. 12, is taken for granted in the *Dioptra*. Secondly, the sides which in the numerical example of the *Dioptra* are 12, 13 and 15 *parts* or *shares* (μοῖραι) become 12, 13 and 15 *times unity* (12, 13 and 15 μονάδες).

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[3] The order of the main chapters, on the other hand, is likely to be changed in at least one case: Hero deals with triangles before trapezia and irregular quadrangles, whereas treatises close to the practical tradition treat of quadrangles in general before triangles, at least outside the Greek orbit; it is not to be seen, however, whether Hero himself or the Greek handbook which he uses is responsible for the inversion.

[4] That this is what happens (unless Hero borrows for the *Metrica* from an original from which he copies even more directly in the *Dioptra*) is argued in [Høyrup 1997b].

[5] That is, the side of its area when it is laid out as a square; in numbers, which is probably what Hero intends, its square root.

The second example has to do with the way theoreticians deal with sub-scientific recreational mathematics. In *Arithmetica* I.22, [ed. Tannery 1893: 50–52] Diophantos asks for three numbers which fulfil the condition that if to each of them a given part of the sum of the others is added, the same sum arises in all cases. There is no doubt that this is an undressed version of the "purchase of a horse", to which already Plato seems to refer in book I of the *Republic* [333b-c, ed., trans. Shorey 1930: I, 332*f*]: Three men go to the market in order to buy a horse; the first asks for (e.g.) half of the possessions of the others, which will enable him to buy the horse; the second asks for one third; and the third is satisfied by a mere fourth. But Diophantos transforms the problem into a pure-number problem without hinting in any way at its origin as a riddle.

Another recreational problem which was no less popular during the Middle Ages and which was probably also known in Classical Antiquity [Christianidis 1991] is "the hundred fowls" – how to buy one hundred fowls for one hundred monetary units, given that (e.g.) a goose costs 3 dinars and a hen 2 dinars, chickens being sold one dinar each three. In his *Book of Rare Things in Calculation*, Abū Kāmil submits this indeterminate problem to a comprehensive mathematical treatment, finding the complete set of solutions; at first, however, he tells [German trans. Suter 1910a: 100] that he is going to treat

> a particular type of calculation, circulating among high-ranking and lowly people, among scholars and among the uneducated, at which they rejoice, and which they find new and beautiful; one asks the other, and he is then given an approximate and only assumed answer, they known neither principle nor rule in the matter.

It is obvious from the wording that Abū Kāmil the scientific mathematician does not approve of such non-scientific manners. But he recognizes the existence of the sub-scientific approach, confronts it, and makes clear that he is taking up one of its favourite problems and submitting it to what he would consider a genuine mathematical treatment.[6]

A third example is offered by the divergent approaches to computation in astronomy.

The preface to book III of the *Almagest* [ed., German trans. Manitius 1912: I, 130] states that books I and II have explained the absolutely indispensable mathematical preliminaries for knowing about the Earth and the Heavens. As it was pointed out by Olaf Pedersen [1974: 32*f*], this means that logistics, the actual computations leading from the theorems of spherical trigonometry (theorems which are dutifully explained because they are not in the *Elements*) to the values of the chords in the chord table of I.11 are not counted as part of mathematics, not even as some kind of related field which in the likeness of mathematics asks for proof.

Also in a preface – namely to the *Book on finding the Chords in the Circle* ... [German trans. Suter 1910b: 11*f*] – , al-Bīrūnī states that

> You know well, God give you strength, for which reason I began searching for a number of demonstrations proving a statement due to the ancient Greeks concerning the division of the broken line in an arbitrary circular arc by means of the perpendicular from its centre, and which passion I felt for the subject [...], so that you reproached [?] me my preoccupation with these chapters of geometry, not knowing the true essence of these subjects, which consists precisely in going in each matter beyond what is necessary. If you would only, God give you strength, observe the aims of

[6] Much more than such recognition is found in Qusta ibn Lūqā's *Treatise on the Proof of the Method of the Double False Position* [ed. Suter 1908]. Qustā offers geometric proofs of the correctness of the method, which he explicitly tells to belong to the "art of reckoning". Similar examples abound.

geometry, which consist in determining the mutual relation between its magnitudes with regard to quantity, and [if you would only observe] that it is in this way that one reaches knowledge of the magnitudes of all things measurable and ponderable found between the centre of the world and the ultimate limits of perception through the senses. And if you would only know that by them [the geometrical magnitudes] are meant the [mere] forms, detached from matter [...]. Whatever way [the geometer] may go, through exercise will he be lifted from the physical to the divine teachings, which are little accessible because of the difficulty to understand their meaning, because of the subtlety of their methods and the majesty of their subject, and because of the circumstance that not everybody is able to have a conception of them, especially not the one who turns away from the art of demonstration. You would be right, Good give you strength, to reproach me, had I neglected to search for these ways [methods], and used my time where an easier approach would suffice; of if the work had not arrived at the point which constitutes the fundament of astronomy, that is to the calculation of the chords in the circle and the ratio of their magnitude to that supposed for the diameter [...].

Only in God the Almighty and All-wise is relief!

The first part of the process – "determining the mutual relation between its magnitudes with regard to quantity, [namely geometrical magnitudes which are understood as mere] forms, detached from matter" – this sounds rather Greek, though more tainted by Greek *philosophy* than by what is found in explicit words in Greek *mathematics*. The second part of the process, however, the application to trigonometric computation, is precisely the step in the process which disappears from view in Ptolemy, as does the actual drawing of the line. What al-Bīrūnī does here is, so to speak, to follow Plato's rule from the *Republic:* that the philosophers who have seen the heavenly light should redescend into the cave to make their insight serve – a rule which the mathematicians of Plato's own cultural orbit were unable or unwilling to comply with.

So far we have moved within the orbit of the exact sciences and only looked at classical antiquity and the Islamic Middle Ages. Similar contrasts turn up in the fourth and final example, which goes to the other end of the spectrum of knowledge and regards the complex of political and economical thought, and which will also give us the occasion to include Latin Europe in the discussion.

Book I of Aristotle's *Politica* consists of economic theory, if economy is taken in its original sense of *oiko-nomia*, "household science". The book is a reflection on theoretical problems: the composition of the *polis* from households, the two-fold composition of the household from male and female and from master and slave; the respective roles of household, village and *polis* for satisfying human needs; the legitimacy of slavery; the distinction between "unnatural" and "natural" property acquisition (the former connected to the invention of coined money and to the possibilities of unlimited accumulation for its own sake that follows from this invention, the latter concerned with the fruits of the earth and animals); the relation between the various kinds of persons involved in the household; and the importance of virtue.

The existence of practical advice and of common-sense anecdotes on gaining wealth is mentioned in chapter 11 – for instance the anecdote on Thales's corner on oil-mills. Such explicit and implicit practical advice, however, is clearly understood [1258^{b}39–1259^{a}6, trans. Jowett 1921] as belonging to another genre than Aristotle's own treatise, a genre that includes

works [...] written upon these subjects by various persons; for example, by Chares the Parian, and Apollodorus the Lemnian, who have treated of Tillage and Planting, while others have treated of other branches [...]. It would be well to collect the scattered stories of the ways in which individuals have succeeded in amassing a fortune; for all this is useful to persons who value the art of getting wealth.

The treatises of Chares and Apollodoros are evidently of the same kind as Columella's *De re rustica* (though shorter), i.e., as deficient when it comes to theory as *Politica* is lacking in practical advice.

Book I of the pseudo-Aristotelian *Oeconomica* does combine two kinds of knowledge, but not theory and real practical advice.[7] E. S. Forster describes it instead as containing "elements derived from Aristotle, but it also owes a good deal to the *Oeconomicus* of Xenophon". The latter work thus seems to represent the kind of "practical" knowledge the followers of Aristotle thought it would be adequate for a gentleman to possess.

Xenophon's approach does not differ much in tone and intellectual level from that other famous conservative gentleman's advice for marriage, "tak'em young and treat'em rough"; no genuine interest in practical problems is to be expected, we might say, in a work based on the opinion that "the illiberal arts, as they are called, are spoken against, and are, naturally enough, held in utter disdain in our states" [iv.2, trans. Marchant 1923: 391], and which claims that what it presents is indeed an ἐπιστήμη, a "science", no art – and at that the science best fit for gentlemen who at all costs should keep aloof of illiberal arts [vi.2–8, *ibid.* p. 409]. Whatever practical advice comes in the end is commonsensical and not the kind of knowledge which the good overseer should possess. It is stated quite explicitly that technical progress is of no use: "It is not the farmers reputed to have made some clever discovery in agriculture who differ in fortune from others: it is things of this sort [i.e., taking trouble to see what is sown or manured, taking trouble to plant wines, etc.] that make all the difference" [xx.5, *ibid.* p. 511]; but this all-important care then receives no closer description.[8]

An obvious contrast to this juxtaposition of theory and banality parading as practical advice is offered by an Arabic handbook on "commercial science" written by one Šaykh Abū'l-Fadl Jaᶜfar ibn ᶜAli al-Dimišqī somewhere between 870 and 1174 CE. This treatise combines general economic theory (about the distinction between monetary, movable and fixed property, more elaborate than Aristotle's conceptual distinctions but on the same lines) and Greek political theory with systematic description of various types of goods and with (really) good advice on prudent trade[9].

Around 1240, finally, Robert Grosseteste – certainly better versed than most of his contemporaries in philosophical writing – prepared or supervised the preparation of a set of *Rules on How to Guard and Govern Land and Households*. The view of the scholar-bishop on how a handbook for practice should be written is not to be mistaken: philosophical doctrines have no role to play, neither as adornment nor as an integrative part of the exposition. The rules are real practical rules, not cant in the manner of a Xenophon. They come close to *De re rustica*

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[7] Book II, mainly a collection of anecdotes, is from a different hand and a different epoch. Book I, on the other hand, seems to have been produced in the Peripatetic school not long after Aristotle's own times – see [Forster 1921, preface].

[8] Certainly not because Xenophon did not possess or appreciate the importance of such cunning – how else should have he have been able to lead the army of the "Ten Thousand" successfully through vast unknown regions to the sea, and how should he have been able to make a career as a mercenary general in territories with no particular feeling for Greek cultural values?

[9] A discussion and a partial translation of the treatise is given by Hellmut Ritter [1916].

and probably to the works of a Chares and an Apollodoros [trans. Oschinsky 1971: 391–395]:

> [In order to ascertain whether you can live throughout the year off your demesne lands,] command emphatically that in each place, when the corn is brought in, the twentieth sheaf of each kind of corn is thrown aside as it enters the grange and is threshed and measured by itself; and based on this measure you can estimate all the other corn in the grange. And when this is being done I advise you to send to the best manors among your lands those men from your household in whom you have the greatest trust so that they be present at harvest time when the corn is brought in and there supervise this operation. And if this does not please you use this method: Command your steward that he arranges for knowledgeable and loyal men to estimate every year at Michaelmas all the stacks, within and without the grange, of each kind of corn; how many quarters there ought to be, and how many quarters in seed-corn and in liveries of corn to servants the soil will take back [...].

No doubt about it, these are stratagems actually used by clever stewards,– stratagems in which (ps.-)Grosseteste advises the landowner to instruct less clever or less experienced stewards.

Overall observations

Since Dilthey, the reasons for the lacking integration of philosophy with technical-practical cognition in the Greek world has been discussed with reference to "the opposition of a ruling citizenry, which also cultivated science, to the slave class on which the burden of manual labor fell, and, connected with that, disdain for physical labor", and to "the lack of an industry managed by scientifically trained people" [Dilthey 1988: 203].

Such explanations, however plausible when seen in the perspective of an epoch where "an industry managed by scientifically trained people" has become a matter of course, were evidently invisible to the actors themselves, according to whom the illiberal arts were, "naturally enough, held in utter disdain". But the resulting attitude – however much a "rationalization" in the Freudian sense – is formulated with perspicacity by Aristotle in the *Metaphysics* [981b14-982a1, trans. Ross 1928]:

> At first he who invented any art whatever that went beyond the common perceptions of man was naturally admired by men, not only because there was something useful in the inventions, but because he was thought wise and superior to the rest. But as more arts were invented, and some were directed to the necessities of life, others to recreation, the inventors of the latter were naturally always regarded as wiser than the inventors of the former, because their branches of knowledge did not aim at utility. Hence when all such inventions were already established, the sciences which do not aim at giving pleasure or at the necessities of life were discovered, [...]
>
> So [...], the theoretical kinds of knowledge [are thought] to be more of the nature of Wisdom than the productive.

The discovery of the "sciences which do not aim at giving pleasure or at the necessities of life", i.e., of the possibility to pursue autonomous knowledge, is what constitutes the illustrious "Greek miracle"; but although writers with a broader outlook than Xenophon – a Plato, an Aristotle – did not stop at contempt, Greek and Hellenistic antiquity in general tended to hypostatize the autonomy of theoretical knowledge; even the practical underpinnings of theoretical investigations like the drawing of lines are excluded from view and only enter the scene by way of their outcomes.[10] Attempts to apply theoretical knowledge in order to improve upon technical practice

[10] Evidently, this concerns what went into written treatises – nobody doubts that Greek geometers made their own drawings. Nor will anybody deny the technical feats of Greek and Roman antiquity – as David

were liable to produce incoherent eclecticism like the Heronian *Metrica*, or to impress irrelevant schemes like the humoral theory on a medical *techne* which without this borrowing from fully developed natural philosophy was more "scientific" in Baconian and probably even in modern terms.[11] As summed up by Benjamin Farrington [1969: 302] (no less a Baconian than a Marxist), it was not

> only with Ptolemy and Galen that the ancients stood on the threshold of the modern world. By that late date they had already been loitering on the threshold for four hundred years. They had indeed demonstrated conclusively their inability to cross it.

It is a commonplace that the contempt for practical – and even manual – labour became much less outspoken in Latin Western Europe. Often the ideological influence of Christianity (embodied in the Rule of St. Benedict) is seen as the cause for this change; one might suggest that a social situation where the carriers of high literate culture were forced to act themselves as overseers both regarding their own affairs and in the affairs of the state may well have shaped Christian ideology quite as much as Christian ideology – after all still in the crucible – acted as an independent force. This would hold in the Early and Central Middle Ages, where monks were forced to manage their own monastic economy and control their own serfs directly, and where Charlemagne had to use the bishops as administrators of the Empire. But it is obvious that Grosseteste's situation was not much different – if he did not have to act directly as an overseer he had at least to guide his steward. He knew, furthermore, that the steward had no use for philosophy but rather for organized common sense and the experience of colleagues and predecessors; Aristotle's observations in another passage from the *Metaphysics* (981^{a}13–23, slightly earlier than the above quotation [trans. Ross 1928]) still held good:

> With a view to action experience seems in no respect inferior to art, and men of experience succeed even better than those who have theory without experience. (The reason is that experience is knowledge of individuals, art of universals, and actions and production are all concerned with the individual; for the physician does not cure *man*, except in an incidental way, but Callias and Socrates or some other called by some other individual name, who happens to be a man. If, then, a man has the theory without the experience, and recognizes the universal but does not know the individual included in this, he will often fail to cure; for it is the individual that is to be cured.)

Scholastics like Grosseteste hence recognized the legitimacy of practical knowledge; since they knew Aristotle much better than Xenophon, this valuation was not even likely to cause

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King once remarked in a discussion of the topic, the Antikythera mechanism [see Price 1974] is another "Greek miracle", to which one may add that the logistics of Alexander's conquests bears witness of a high level of practical economical thought, much more sophisticated than what the soldier and former general Xenophon puts into writing.

Yet what is put into writing *and what not* reflects very well what the culturally hegemonic stratum finds it suitable to think. However oblique their relation to actual practice, Greek (and Islamic and Medieval Latin) writing remains an adequate gauge of prevailing attitudes to the relation between different types of knowledge.

[11] No doubt skilled physicians were able to formulate empirically gained insights within the humoral framework; if that had not been the case, even the most skilful strategies of the medical professions would not have been sufficient to make the theory survive into the nineteenth century CE. But the basic theoretical framework itself remains a purely speculative (Latin / English translation of *theoretical!*) calque from the doctrine of the four elements.

them any major trouble; but they had no incentive to produce a genuine integration for which their theory was not really fit; when they tried, the result turned out [Beaujouan 1957: 20] to be characterized by

> une recherche trop systématique de l'utilité immédiate et par un attachement trop servile aux données du sens commun

– "trop" at least with regard to our aspiration to reach the birth of modern science, but perhaps not in relation to the practical concerns of the day, since these were not likely to be adequately served by much of the theory at hand.

Attempts to achieve genuine integration are only found in the Islamic world as it unfolded in the ninth through twelfth centuries CE (the situation during the ensuing period, characterized by what A. I. Sabra [1987: 240] labels "naturalization", is different and should probably be investigated in other terms). Abū'l-Wafāʾ's recognition of the legitimacy of the practical perspective coupled to awareness of the potentialities of adapted theory within the practical context is characteristic, and also reflected in the text on commercial science and in the innovative use of natural philosophy in the medical writings of al-Razī and ibn Sīnā. The above quotation from al-Bīrūnī shows the underlying attitude: systematic thought is obliged to strive for the highest theoretical insight, but *also* to "return to the cave". At an earlier occasion [Høyrup 1987: 300*ff*] I have pointed to the parallel between this attitude and the religious base of the Islamic culture: Allāh, supremely transcendental, is still Providence caring for the smallest details, and much more so than the Christian God(s), who had delegated much of his/their care for everyday to the Virgin and the saints.[12] As long as the inherent fundamentalism of Islam was not controlled by a priesthood, it was possible for an al-Bīrūnī to judge for himself how *his* service to the divine order was to be made, and for a geometer to decide that *his* service was the mathematical determination of the direction of prayer irrespective of the fact that religious scholars had their own ways[13] and did not care about spherical geometry.

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[12] The example of Qustā ibn Lūqā shows that the influence of the religious tenets of Islam, if it is indeed present, was mediated by the general cultural climate of the Islamic culture as it developed after the ʿAbbasid revolution, and was never direct: there is no difference in attitude between the Christian Qustā and the Muslim ibn al-Haytham when they submit the ways of practical reckoning to mathematical scrutiny, and the difference between Qustā and Abū Kāmil is one of personal style and not of creed.

[13] Ways that might depend on "the sun, the stars, and even the winds", and which might make one law-school in Samarqand favour the south and another the west [King 1982: 304*f*, quotation p. 304].

Wheatsheaf.

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3. Latin Pedagogical Philosophies and Practices.Â While chapter two considers the theory of language learning, chapters three and four will focus on the practicalities of Latin teaching. In order to gain a more thorough understanding of the varying experiences of Latin teachers and the challenges they face, I circulated a survey regarding teaching practices and philosophies through popular social media groups for Latin educators.Â See Bonner, Education in Ancient Rome, chapter 3 and Marrou, A History of Education in Antiquity for more on Roman education. 12 Quintilian also justifies teaching Greek first since Latin derives from Greek. Quint. The Islamic view of nature during the Golden Age had its roots in the Quran, the very word of God and the basis of Islam. Muslim scholars at that time were inspired to study nature in the context of the Quran.Â These institutions were far superior to any that existed in ancient times or in lands beyond the Islamic Empire. In medieval Europe most hospitals were attached to religious orders and monasteries.Â Muslims derived their theory of numbers (â€˜ilm al-aâ€˜dad) in arithmetic from translations of the Greeks sources such as Books VÎ™Î™ through to Î™X of Euclidâ€™s Elements and the Introduction to the Science of Numbers by Nicomachus of Gerasa (Berggren, 1997). Moreover, they acquired numerals from India (Hindu) and possibly China and made their use widespread. Â· Islam spread rapidly throughout the Middle East and beyond. Science and technology in Medieval Islam. Science and technology in Medieval Islam. Early Islamic teaching encouraged and promoted the pursuit of scholarship and science.Â Astronomy was important to Muslims for very practical and religious reasons: Astronomy aided navigation for purposes of trade and travel, and it was important in determining an accurate lunar calendar, prayer times and the direction of Mecca. Important Islamic observatories were established in many cities across the Islamic world in order to make accurate observations of the sun, moon and stars. Accurate calendars were important to determine religious festivals such as the period of fasting known as Ramadan. In ancient, islamic. And medieval latin contexts. Jens H0yrup. As all contributors to the present volume know, Laura Toti Rigatelli.Â Similar examples abound. Theory and practice in ancient contexts. theorems of spherical trigonometry (theorems which are dutifully. explained because they are not in the Elements) to the values of the.