

Short Course: Multiscale statistical approach with applications in astronomy and solar physics

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Abstract

The objective of this short course is to study several multiscale statistical tools and to show how they are used to solve practical problems arising in astrophysics and in solar physics. After having motivated the use of multiscale approach in astronomy, we study in more details a few problems.

Detection of Gamma Ray Bursts We first present the Haar wavelet transform, and see how it can be used to detect Gamma Ray Bursts (GRB). These are commonly associated with star formations and supernovae, but their precise origin is still unknown. The intensity profile underlying Gamma Ray Bursts can be modeled as an inhomogeneous Poisson process observed on a background made of Poisson noise. We study the method proposed by Kolaczyk [3] that aims at removing Poisson noise. This procedure is based on the Haar transform and on a thresholding scheme specifically tailored to Poisson noise.

Analysis of solar images and introduction to fractal models The discrete wavelet transform (such as the Haar transform) is useful to *denoise* signals. However, when the aim is to *analyze* images, it is sometimes useful to consider the Continuous Wavelet Transform, since it gives information about the signal at all desired scales and locations. The Extreme ultraviolet Imaging Telescope (EIT) on board the SoHO mission provides since 1996 images of the solar atmosphere in four different passbands. It is of interest to analyze these images using signal processing methods in order to, e.g., automatically detect solar events. In the second course, we show how the scale measure associated to the Mexican Hat wavelet transform allows to build pertinent times series from a solar images dataset. We describe three applications of the use of the scale measure as presented in [1]. Finally, we introduce fractal models and show their potential to emulate images of the solar corona.

Multiscale polynomial estimation The third course will be devoted to a new non-parametric methodology introduced in [5] that relies on a *multiscale* signal decomposition based on *polynomials*. The corresponding estimator uses a penalized likelihood, and may be tailored to estimate either the intensity function of a Poisson process or a density. We show some optimality properties of this estimator.

Detection of clusters in a galaxy We show how nonparametric kernel density estimation allows to separate different clusters in a galaxy. Indeed, the presence of a cluster in a data sample is indicated by a peak in the probability function underlying the data. The method presented in [4] provides a kernel density estimator adapted to this problem, and tests for the presence of modes in the density function. We compare this method with other procedures described in [2].

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6 Further readings

- J.-L. Starck and F. Murtagh. *Astronomical Image and Data Analysis*. Springer, 2002.
- Rebecca Willett. Multiscale analysis for intensity and density estimation. Master's thesis, Rice University, 2002.

References

- [1] V. Delouille, J. de Patoul, J.-F. Hochedez, L. Jacques, and J.-P. Antoine. Wavelet spectrum analysis of solar EUV images: method and applications to network characteristic scale evolution, flare nowcasting, and extraction of active regions with EIT/SoHO. Submitted to *Solar Physics: Topical issue for Signal Image processing*. Available at <ftp://omaftp.oma.be/pub/astro/verodelo/ScaleMeasure-Delouille-VF1.ps>, February 2005.
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- [5] R. Willett and R. Nowak. Multiscale likelihood analysis and image reconstruction. In *Wavelets X, SPIE meeting*, 2003. Available at <ftp://omaftp.oma.be/pub/astro/verodelo/Willett2003.pdf>

Solar Physics, Astrophysics and Astronomy. Space Plasma Physics. Space Weather. The probabilistic approach we advocate overcomes these problems in a natural way with two significant improvements over existing approaches. First, we use all available observations, weighted by their uncertainties. Ideally, we would rely on time-dependent uncertainties provided by the instrument teams. Possible trends between solar minima are too weak to be statistically significant except for the downward trend between 1996 and 2009. Solar images taken simultaneously at different wavelengths in the EUV are widely used for understanding structures such as arcs, coronal holes, loops, etc. The line-of-sight integration and the finite spectral resolution of EUV telescopes, however, hinders interpretation of these individual images in terms of temperature bands. Gaussian images, however, tend to be the exception in astronomy; the departure from a Gaussian PDF is precisely exploited by the ICA to improve the separation of the sources. This will be illustrated shortly in Sect. Numerous successful applications of ICA to blind deconvolution and to hyperspectral imaging have been reported. Several variants have also been proposed to tailor the method to physical constraints, see for example Hyvärinen et al. Astronomy is the science devoted to the study of planets, stars, galaxies and the Universe as a whole. It is closely coupled to astrophysics where laws governing physical processes (e.g., gravity, electromagnetism, quantum mechanics) are used to model observed astronomical properties. As the oldest physical science, astronomy has served the development of statistical methodology for centuries. In the past century, as astronomy turned to physics for insights and statisticians turned to applications in human sciences and industries. In the past two decades, the cross-disciplinary field of astrostatistics has reemerged to deal both with important astrophysical issues and to treat mega-datasets produced by high-technology observatories. Deconvolution of astronomical images using the multiscale maximum entropy method. *Astronomy and Astrophysics, Suppl. Ser.* , 315:575–585, 1996. CrossRefADSGoogle Scholar. [SBLP94] J.L. Starck, A. Bijaoui, B. Lopez, and C. Perrier. Image reconstruction by the wavelet transform applied to aperture synthesis. *Astronomy and Astrophysics* , 283:349–360, 1994. ADSGoogle Scholar. [SBVM00] J.L. Starck, A. Bijaoui, I. Vachanov, and F. Murtagh. A combined approach for object detection and deconvolution. *Astronomy and Astrophysics, Suppl. Ser.* , 147:139–149, 2000. The journal covers solar physics, planetary systems, stellar, galactic and extra-galactic astronomy and astrophysics, as well as cosmology. *New Astronomy Reviews* is also open for proposals covering interdisciplinary and emerging topics such as astrobiology, astroparticle physics, and astrochemistry. Visit the journal homepage for full aims and scope, to submit your articles or subscribe. *Physics of the Earth and Planetary Interiors*. Launched in 1968 to fill the need for an international journal in the field of planetary physics, geodesy and geophysics, *Physics of the Earth and Planetary Interiors* has now grown to become important reading matter for all geophysicists. It is the only journal to be entirely devoted to the physical and chemical processes of planetary interiors.