ON MANAGING CENTRAL BRANCH RISK NETWORKS, USING SCALE FUNCTIONS

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Abstract

The study of the interaction between a reinsurer and several insurers, or between a central branch (CB) and several subsidiary companies suggest the concept of CB risk networks, and many interesting problems.

Keywords: risk networks, valuation problem, dividends optimization, liquidation penalty, spectrally negative processes, scale functions

1 Central branch risk networks

Definition 1. **A central branch (CB) risk network** *is formed from:*

1. *Several spectrally negative subsidiaries* $X_i(t), i = 1, \ldots, I$, *which must be kept above certain prescribed levels* o_i *by bailouts from a central branch (CB)* $X_0(t)$, *or be liquidated when they go below* o_i.

2. *The reserve of the CB is a spectrally negative process denoted by* $X_0(t)$ *in the absence of subsidiaries, and by* $X(t)$ *after subtracting the bailouts. The ruin time*

$$\tau = \tau_0^- = \inf\{t \geq 0 : X(t) < 0\}$$

causes the ruin of the whole network and leads to a severe penalty.

3. *The CB must also cover a certain proportion* $\overline{\alpha}_i = 1 - \alpha_i$ *of each claim* $C_{i,j}$ *of subsidiary* i, *leaving the subsidiary to pay only* $\alpha_i C_{i,j}$, *where* $\alpha_i \in [0, 1]$ *are called proportional reinsurance retention levels.*

Remark 1. *For a CB network, the boundaries* $u_i = o_i, i = 1, ..., I$ *are reflecting and* $u_0 = 0$ *is absorbing.*

Two important characteristics of a CB network are:

1. the ruin probability transform:

$$S_q(\boldsymbol{u}, \boldsymbol{\theta}) := E_{\boldsymbol{u}} \left[e^{-q\tau + <\boldsymbol{\theta}, \boldsymbol{X}(\tau)>}; \tau < \infty \right],$$

$$\boldsymbol{X}(t) = (X_0(t), X_1(t), ..., X_I(t)). \quad (1)$$

2. the optimal discounted dividends until ruin:

$$V^F(\boldsymbol{u}) := \sup_{\pi=(R_0, R_1, \ldots, R_I)} E_{\boldsymbol{u}} \int_0^\tau e^{-qt} \left(\sum_{i=0}^I dR_i(t) \right),$$

where R_i denotes the nonnegative cumulative dividends/consum
process paid by the i-th branch.

More generally, τ could be replaced by other stopping times, like the drawdown time

$$\tau_\xi := \inf\{t \geq 0 : X_t \leq \xi \sup_{0 \leq s \leq t} X_s\},$$

where $\xi \in (0, 1)$ is a fixed constant.

1.1 Heuristic valuation using equilibrium line policies

A natural approach for **evaluating financial companies**, going back to de Finetti [DF57] and Modigliani and Miller [MM61] is to consider the optimal expected discounted cumulative dividends/optimal consumption until ruin (2) – see [LST14] for further references on this venerable approach.

If the liquidation time τ is also optimized

$$\mathcal{I}^F(\boldsymbol{u}) := \sup_{\pi=(R_0,R_1,\ldots,R_I,\tau)} E_{\boldsymbol{u}} \int_0^{\tau} e^{-qt} \left(\sum_{i=0}^{I} dR_i(t) \right), \quad (3)$$

the result $\mathcal{I}^F(\boldsymbol{u})$ is a Gittins type valuation index. We will propose now a heuristic multi-dimensional valuation index inspired by the remarkable fact that central branch network problems admit occasionally explicit answers, if the retention levels are small enough [APP08a, APP08b, BCR11, AMP16].

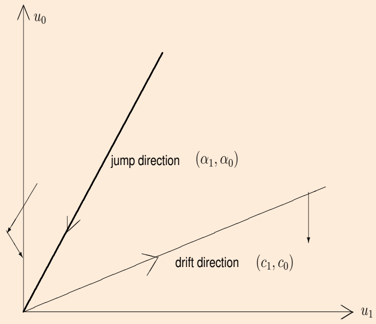

Figure 1: Geometrical considerations

We recall from the papers above that when $I = 1$ and

$$c_0 \leq c_1 \frac{\overline{\alpha}_1}{\alpha_1},$$

i.e. if the angle of the vector $\boldsymbol{\alpha} = (\alpha_1, \overline{\alpha}_1)$ with the u_1 axis is bigger than that of $\boldsymbol{c} = (c_1, c_0)$, then the lower cone

$$\mathcal{C} := \{0 \leq u_0 \leq u_1 \frac{\overline{\alpha}_1}{\alpha_1}\}$$

contains $\boldsymbol{c}$ and is invariant with respect to the stochastic flow, and the ruin probability is a classic one-dimensional ultimate ruin probability

$$\Psi(u_1, u_0) = \Psi(\alpha_1 \frac{u_0}{\overline{\alpha}_1}, u_0) := \Psi_0(u_0), \forall u_0$$

Turning now to several dimensions several dimensions, it is easy to check that:

Lemma 1. *The cone*

$$\mathcal{C} := \{0 \leq u_0 \leq u_i \frac{\overline{\alpha}_i}{\alpha_i},\ \overline{\alpha}_i = 1 - \alpha_i,\ i = 1, ..., I\}$$

is invariant under the "(extra) cheap reinsurance" condition

$$c_0 \leq c_i \frac{\overline{\alpha}_i}{\alpha_i}, i = 1, ..., I. \tag{4}$$

The boundary edge

$$u_1 \frac{1 - \alpha_1}{\alpha_1} = ... = u_i \frac{1 - \alpha_i}{\alpha_i} = u_0,\ i = 1, ..., I, \tag{5}$$

to be called "**equilibrium line**", plays a prominent role in two recent papers, [BB11] [§] and [AMP16], who solved the optimal dividends problem in the (extra) cheap reinsurance two-dimensional case $c_1 \frac{1-\alpha_1}{\alpha_1} > c_0$. The last paper showed that:

[§] who computed an explicit value function maximizing an expected exponential utility at a fixed terminal time for multi-dimensional reinsurance model under the "cheap reinsurance" assumption that the drifts point along the line $c_1 \frac{1-\alpha_1}{\alpha_1} = ... = c_I \frac{1-\alpha_I}{\alpha_I}$.

1. Starting from the equilibrium line, the optimal policy is to stay on this line by cashing the excess income of the subsidiary as dividends.

2. Starting from points away from the equilibrium line, in the cheap reinsurance case, the optimal policy is to reach the equilibrium line by one lump sum payment.

3. In the extra cheap reinsurance case, the optimal policy is more complicated, when starting in a certain egg-shaped subset of the non-invariant cone (where parts of the premia are cashed, following a ”shortest path”, in some sense).

The first two findings prompt us to introduce multi-dimensional ”equilibrium line” policies for (extra) cheap reinsurance networks, under which the network follows this line in the absence of claims, by **subsidiaries cashing part of their premia as dividends**. Subsequently, whenever the CB or one subsidiary drop below, **all the other subsidiaries** reduce their reserves by **lump sum dividend** taking, bringing back the process on the equilibrium line.

Remark 2. *These strategies may not be optimal; however, since they stipulate that the subsidiaries just ”***accompany deterministically***” the CB, the value of the network expected dividends decomposes as a sum of one-dimensional quantities– see next Lemma.*

Lemma 2. *For a general CB network, and a fixed admissible dividends process $R_0(t)$, the de Finetti value function for the equilibrium policy associated to $\pi = (R_0, \tau)$ is:*

$$V_\pi^F(x) = E_x \left[\int_0^\tau e^{-qt} \Big[dR_0(t) + \widetilde{c} dt - \gamma dX_0(t) - \sum_{i=1}^{I} (\gamma \frac{\overline{\alpha}_i}{\alpha_i} - 1) dX$$

where

$$\gamma = \sum_{i=1}^{I} \frac{\alpha_i}{\overline{\alpha}_i}, \ \widetilde{c} = \gamma \sum_{i=1}^{I} c_i \frac{\overline{\alpha}_i}{\alpha_i}.$$

Optimizing dividends reduces thus to a one-dimensional problem.

2 Valuation of spectrally negative Lévy subsidiaries using dynamic indices built from the scale functions

Consider a subsidiary with liquidation value $w(x)$, and the value function of a policy $\pi = (R, R_*, \tau)$ involving some dividend process R, bailout process R_*, and stopping time τ. Modify now this value function by subtracting a constant subsidy $\mathcal{I} = \mathcal{I}(x)$ for stopping, and choose this subsidy so that the decision of whether to continue or stop yield equal payoffs. Suppose that $dR_*(t)$ consists of a single payoff at the liquidation/reevaluation time τ.

Then, the valuation index is provided by the equation:

$$\sup_{\pi} E_x \Big[\int_0^{\tau} e^{-qt} \Big(dR(t) + dR_*(t) \Big) \Big] - \mathcal{I} = w(x)$$

$$\Longrightarrow \mathcal{I}(x) = \sup_{\tau} E_x \left[\int_0^{\tau} e^{-qt} dR(t) + e^{-q\tau} w(X(\tau)) \right] - w(x)$$

As a simplification of this optimal stopping problem, suppose that the stopping time is prescribed by forced stopping, for example at $\tau = \tau_o^-$.The result is a modified De-Finetti objective

$$V^F(x) = E_x \left[\int_0^{\tau} e^{-qt} dR(t) + e^{-q\tau} w(X(\tau)) \right] - w(x) \quad (6)$$

With linear liquidation costs $w(x) = \begin{cases} kx - K, & x < 0 \\ x - K, & x \geq 0 \end{cases}$, where $K \geq 0$ is a penalty for quick liquidation, and $\tau = \tau_0^-$, this problem was essentially studied in [LR10, APP15]. The approximate index is then:

$$V^F(x) = \mathcal{S}_w(x) + W_q(x) \frac{1 - \mathcal{S}'_w(b)}{W'_q(b)} - w(x), \tag{7}$$

$$\mathcal{S}_w(x) = kZ_{1,q}(x) - KZ_q(x), \; Z_{1,q}(x) = \overline{Z}_q(x) - p\overline{W}_q(x)$$

The optimal b is typically the last maximum of the barrier function

$$G(b) = \frac{1 - \mathcal{S}'_w(b)}{W'_q(b)}. \tag{8}$$

The optimization of the bailout point o has been less studied, and deserves further attention.

Remark 3. *Several variations of this index may be obtained replacing absorption at τ by Parisian absorbtion or reflection, and by adding refraction or other boundary mechanisms; the valuation index $\mathcal{I}$ is again given by* (7)*, once one uses the appropriate scale functions $W, Z, \mathcal{S}_w$ [KL10, AIZ14, APP15, APY16].*

3 Eight first passage laws for Poissonian (Parisian) detection of insolvency

A useful type of models developed recently [AIZ14, AI15, APY16] assume that insolvency is only **observed periodically**, at an increasing sequence of *Poisson observation times* $\mathcal{T}_r = \{t_i, i = 1, 2, ...,$ the arrival times of an independent Poisson process of rate r, with $r > 0$ fixed.

The analog concepts for first passage times are the stopping times

$$T_b^+ = \inf\{t_i : \ X(t_i) > b\}, \quad T_a^- = \inf\{t_i > 0 : \ X(t_i) < a\} \quad ($$

A **spectrally negative Lévy processes with Parisian reflection** below 0 may be defined by pushing the process up to 0 each time it is below 0 at an observation time T_i.

Remark 4. *Parisian detection below 0 is related to the "time spent in the red"*

$$T_{<0} := \int_0^\infty I_{\{X(t)<0\}} dt.$$

$$P_x[T_0^- = \infty] = P_x[T_{<0} < \mathcal{E}(r)] = E_x\left[e^{-rT_{<0}}\right] = p\frac{\Phi_r}{r} Z(x, \Phi_r$$

Proposition 1. *Let X be a spectrally negative Lévy process with Parisian detection below 0 at rate r. Then, for $x \in [0, b]$:*

1. *The* **capital injections/bailouts law for a reflected process, until τ_b^+ [APY16, Cor 3.1 ii)]**, *[IP12, Thm 2]. Let $X^{[0}(t)$ denote the SNMAP process reflected at 0, let $R_*(t) = -(0 \wedge \underline{X}(t))$ denote its regulator at 0, so that $X^{[0}(t) = X(t) + R_*(t)$, and let $\mathbb{E}_x^{[0}$ denote expectation for the process reflected at 0. Then:*

$$L_{*,\theta}(x,b) := \mathbb{E}_x^{[0}[e^{-q\tau_b^+ - \theta R_*(\tau_b^+)}] = \begin{cases} Z_{q,r}(x,\theta) Z_{q,r}(b,\theta)^{-1} & \theta < \\ \mathbb{P}^{[0}[\tau_b^+ < T_0] = Z_{q,r}(x,\Phi_{q+r}) Z_{q,r}(b,\Phi_{q+r})^{-1} & \\ := W_{q,r}(x) W_{q,r}(b)^{-1} & \theta = \end{cases}$$

where

$$Z_{q,r}(x,\theta) = \frac{r}{q+r-\kappa(\theta)} Z_q(x,\theta) + \frac{q-\kappa(\theta)}{q+r-\kappa(\theta)} Z_q(x,\Phi_{q+r}),$$

with $\theta = \Phi_{q+r}$ interpreted in the limiting sense.

When $r \to \infty$, $Z_{q,\infty}(x,\theta) = Z_q(x,\theta)$, $W_{q,\infty}(x) = W_q(x)$ and (10) *reduces to classic results [IP12].*

2. *The* **expected discounted dividends until T_0 [AIZ14, (27)]** *are :*

$$V_{q,r}(x,b) = \mathbb{E}_x^{|0,b]}\left[\int_0^{T_0} e^{-qt} dR(t)\right] = W_{q,r}(x) W'_{q,r}(b)^{-1}, \tag{10}$$

where $\mathbb{E}^{|0,b]}$ denotes the law of a process reflected from above at b with Parisian absorbtion at 0, and $R(t)$ denotes the upper regulation at b.

3. *The* **expected discounted dividends with reflection at 0 at Parisian times, until the total bail-outs surpass an exponential variable $\mathcal{E}_\xi$** *satisfy*

$$V^S_{*,\xi}(x,b) = \mathbb{E}_x^{[0,b]}\left[\int_0^{\infty} e^{-qs} 1_{[R_*(s)<\mathcal{E}_\xi]} dR(s)\right]$$
$$= Z_{q,r}(x,\xi) Z'_{q,r}(b,\xi)^{-1}$$

see [AI14, (15)].

4. *The* **severity of Parisian ruin with absorbtion at τ_b^+ [AIZ14, (15)]** *[IP12, Cor 3] is:*

$$S^b(x,\theta) = \mathbb{E}_x\left[e^{\theta X(T_0)}; 1_{T_0<\tau_b\wedge\mathcal{E}_q}\right] =$$
$$Z_{q,r}(x,\theta) - W_{q,r}(x) W_{q,r}(b)^{-1} Z_{q,r}(b,\theta).$$

5. *The* **expected total discounted bailouts at Parisian times up to** τ_b^+ *are given for* $0 \le x \le b$ *by [APY16, Cor 3.2 ii)]:*

$$V_*(x,b) := \mathbb{E}_x^{[0} \left[\int_0^{\tau_b^+} e^{-qt} \mathrm{d}R_*(t) \right]$$
$$= Z_{q,r}(x) Z_{q,r}(b)^{-1} \mathcal{S}(b) - \mathcal{S}(x).$$

where

$$\mathcal{S}(x) = \mathcal{S}_{q,r}(x) = \frac{r}{q+r} \Big(\overline{Z}_q(x) + \frac{\kappa'(0_+)}{q} \Big). \tag{11}$$

6. *The* **total discounted bailouts at Parisian times over an infinite horizon**, *with reflection at* b *are [APY16, Cor 3.4]:*

$$V_*^S(x,b) = \mathbb{E}_x^{[0,b]} \left[\int_0^{\infty} e^{-qt} \mathrm{d}R_*(t) \right]$$
$$Z_{q,r}(x) Z'_{q,r}(b)^{-1} \mathcal{S}'(b) - \mathcal{S}(x).$$

7. *The* q**-resolvent of doubly absorbed Lévy processes** *may be expressed, in terms of the scale function [BPPR, Thm 2]. Namely, for any Borel set* $B \in [a,b]$,

$$\mathbb{E}_x \Big(\int_0^{\tau_a^- \wedge \tau_b^+} e^{-qt} 1_{\{X_t \in B\}} \mathrm{d}t \Big)$$
$$= \int_a^b 1_{\{y \in B\}} \Big[\frac{W_{q,r}(x-a) W_{q,r}(b-y)}{W_{q,r}(b-a)} - W_{q,r}(x-y) \Big] \mathrm{d}y$$

8. *The* **dividends- penalty law for a process reflected at b, with Parisian ruin [AIZ14, (23)]**, *[IP12, Thm 6] is:*

$$S^b_\vartheta(x,\theta) := \mathbb{E}_x^{[0,b]}\left[e^{-\vartheta R(T_0)+\theta X(T_0)}; T_0 < \mathcal{E}_q\right] = Z_{q,r}(x,\theta$$
$$W_{q,r}(x)\left(W'_{q,r}(b) + \vartheta W_{q,r}(b)\right)^{-1}$$
$$\left(Z'_{q,r}(b,\theta) + \vartheta Z_{q,r}(b,\theta)\right).$$

When $x = b$, we may factor the transform (**??**)*:*

$$\mathbb{E}_b^{[0,b]}\left[e^{\theta X(T_0)-\vartheta R(T_0)}; T_0 < \mathcal{E}_q\right] = \Omega(\Omega+\vartheta)^{-1}$$
$$\left(Z_q(b,\theta) - \Omega^{-1}\left(\theta Z_q(b,\theta) + (q-\kappa(\theta))\,W_q(b)\right)\right)\frac{r}{r+q-$$

where

$$\Omega = V^F_{q,r}(b,b)^{-1} = W'_{q,r}(b)W_{q,r}(b)^{-1}$$

By (12)*, $R(T_0)$ and $X(T_0)$ are independent when starting from b, and the former has an exponential distribution with parameter Ω [AIZ14, (23)].*

4 Acceptance-rejection of Lévy subsidiary companies observed at Poissonian times, based on readiness to pay dividends

Even in the one- dimensional case, the final choice of an acceptance-rejection principle is not at all obvious. A first intuition is that an acceptable subsidiary must satisfy the classic positive profit condition

$$p := E_0[X(1)] > 0 \tag{12}$$

or its extension with linear bailout costs [LR10].

Note that the profitability/viability condition of [LR10] is equivalent to

$$G(0) \geq 0,$$

where G is the barrier influence function, and interesting variations may be obtained by replacing absorbtion at 0 with reflection or Parisian reflection, which change the scale functions. The simplicity of all the resulting formulas comes from the fact that the scale functions are only evaluated at 0. This suggested an acceptance-rejection criteria introduced in [AM15, AM16], based on the readiness of subsidiaries to pay dividends at $b = 0$.

Definition 2. *A subsidiary will be called* **efficient** *if the barrier $b = 0$ is locally optimal for paying dividends over some interval $b \in [0, \epsilon), \epsilon > 0$, i.e. if it holds that*

$$G'(0) \leq 0.$$

The motivation of this condition is that companies satisfying it are functional even in the absence of cash reserves, and can contribute cash-flows to the central branch without having to wait first until their reserves build out; efficiency is thus translated as **readiness to pay dividends**. This criterion turns out to be a useful complement of the **viability** concept $G(0) \geq 0$ (which at its turn generalizes the classic $p \geq 0$).

The next result provides a nontrivial efficiency criteria under the SLG infinite horizon cumulative dividends-bailouts objective with Parisian reflection.

Theorem 1. *a) The SLG value function with Parisian reflection and linear bailout costs kx is:*

$$V_{SLG}(x) = Z_{q,r}(x) Z'_{q,r}(b)^{-1} - k \left(Z_{q,r}(x) Z'_{q,r}(b)^{-1} \mathcal{S}'(b) \right.$$
$$= k\mathcal{S}(x) + Z_{q,r}(x) \frac{1 - k\mathcal{S}'(b)}{Z'_{q,r}(b)}$$

b) The barrier $b = 0$ is a local maximum iff the influence function $G(b) := \frac{1-k\mathcal{S}'(b)}{Z'_{q,r}(b)}$ satisfies

$$G'(0) \leq 0 \Leftrightarrow k\Big(\mathcal{S}'(0) Z''_{q,r}(0) - \mathcal{S}''(0) Z'_{q,r}(0)\Big) \leq Z''_{q,r}(0)$$
$$\Leftrightarrow k \leq (1 + \frac{q}{r}) \frac{\Phi_{q+r} - rW_q(0_+)}{\Phi_{q+r} - (r+q)W_q(0_+)}. \tag{13}$$

In the finite variation case § (13) *holds iff*

$$k \leq k(q,r) := (1 + \frac{q}{r}) \frac{\Phi_{q+r} - r/c}{\Phi_{q+r} - (r+q)/c}$$

Remark 5. *It may be checked that $k(q,r)$ increases in q from $k(0,r) = 1$ to infinity and thus an inefficient subsidiary with high transaction cost $k > k(q,r)$ may be turned into efficient by increasing the killing q sufficiently.*

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§in the infinite variation case, the first equation still holds, but the efficiency index does not reflect the distribution, since Φ_{q+r} cancels

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On Central Branch/Reinsurance Risk Networks: Exact Results and Heuristics. by. Florin Avram.Â Modeling the interactions between a reinsurer and several insurers, or between a central management branch (CB) and several subsidiary business branches, or between a coalition and its members, are fascinating problems, which suggest many interesting questions. Beyond two dimensions, one cannot expect exact answers. 5 Central branch networks. 19. 6 Minimizing the ruin probability under transaction costs for linear networks: reduction to one dimension. 20.Â The idea of constructing solutions out of basic scale functions came nally to full bloom while being extended to spectrally negative Markov additive processes (which include LÂ´evy processes in a modulated Markovian environment, and the case of Sparre-Andersen processes with phase-type arrivals) in [KP08, Iva11, IP12], reviewed in the appendix.Â Recently, these tools were used in the context of managing simple risk networks in [AM15, ?], most. notably for judging the eciency of a company which is member of a coalition. The classic De Finetti objective maximizing expected discounted dividends until the ruin. Branch Risk Assessment template: This consists of a standard list of risks, mitigations and KPIs to consider for your branch assessment. Use these as a baseline to get a sense for feasibility and risk appetite and ensure that youâ€™re monitoring performance over time. Copy these from plan to plan to ensure consistency across assessments, while allowing for unique risks, controls or monitoring activities per branch. â€œSet it and forget itâ€ cadence: LogicManager allows you to create an automated, recurring task to update the risk assessments at each of your branches so that you can always be confident that you have the most accurate, up to date information from each branch.