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Some Remarks about General Relativity Principle

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Abstract

In most textbooks that introduce the General Relativity (GR), it is often written that this theory is a generalization of Special Relativity (SR) in which Einstein extends the principle of relativity to all motions. Students who specialize in another branch of physics or mathematics, even after degree did not understand that general covariance is profoundly different from the special relativity principle. This short note reviews this fundamental difference that is, in our opinion, all too often not analyzed in depth. We emphasize that it can be didactically very useful for students to realize how the request of covariance does not necessarily imply a principle of relativity.

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1 Introduction

Let us repeat the words of Salviati in Dialogue Concerning the Two Chief World Systems "The fish in their water will swim toward the front of their bowl with no more effort than toward the back, the butterflies and flies will continue their flights indifferently toward every side and if smoke is made by burning some incense, it will be seen going up in the form of a little cloud, remaining still and moving no more toward one side than to the other". Everything happens as if the ship were not moving at all and certainly Galileo Galilei was the founder of the principle of relativity. After nearly three centuries, Einstein extended, in SR, the Galilean relativity principle to the whole physics. It is not possible to determine the absolute motion of an inertial frame and there is no physical way to differentiate between a reference frame moving at a constant speed and an immobile reference frame. The uniform motion is relative and it can not be perceived except in reference to an external point. Velocity is a relative physical quantity and this is the core of the relativity principle. Therefore in SR, whether or not an object possesses velocity depends on frame of reference. Contrariwise acceleration is absolute and whether or not an object possesses acceleration does not depend on frame of reference. Newton, in his Philosophiae Naturalis Principia Mathematica, tried to show that the acceleration is absolute. Indeed in his famous example of the rotating bucket filled with water, he demonstrates that the centrifugal forces arise only when the water is in absolute rotation with respect to the absolute space. George Berkeley in his De Motu and later Mach proposed a deep criticism of Newton's thought[1]. In particular Mach concluded that the inertia would be an interaction that requires other bodies to manifest itself and it has no sense in a Universe consisting of only one mass [2]. Therefore the concept of absolute motion should be substituted with a totally relative motion. The idea of Mach played an important role in the development of GR. Einstein, following Mach, wanted to make the acceleration a relative magnitude. The starting point was the equivalence between inert mass and gravitational charge and this equivalence principle allowed him to create a new theory of gravity that revolutionized our thinking about the nature of space and time. Gravity is simply geometry of spacetime and the gravitational force becomes a metric force, resulting from the local curvature of spacetime and above all cosmology has been enclosed in the physical disciplines and not only in the philosophical ones when the field equations were applied to the whole universe. All this is wonderful and the theory is now supported by many experimental tests, but none longer ask if Einstein was able to create a fully relativistic theory.

2 The Equivalence Principle

The equivalence principle emerged in the late 16th and early 17th centuries, when Galileo expressed experimentally that the acceleration of a test mass due to gravitation is independent of the amount of mass being accelerated. We have the well know relation

$$\overrightarrow{a} = \overrightarrow{g} \frac{m_G}{m_i} \tag{1}$$

If $\frac{m_G}{m_i}$ is a constant independent of the body, the acceleration is the same for all body. There is no a priori reason, in Newtonian physics, why inert and gravitational masses should be equal but they are found to be identically so, to extremely high precision[3]. Newton itself tested the equivalence principle studying the motion of pendulum of different composition and equal lenght, finding no difference in their periods and from Eötvös to present day the experiments are very accurate and therefore we have, without any doubt, $m_i = m_G$ [4].This formulation is called the Weak Equivalence Principle and it implies that it is impossible to distinguish between the effects of an uniform gravitational field from those experienced in an uniformly accelerated frame, using the simple observation of the free-falling particles behavior. The previous equivalence, extended to all physical laws, is known as the Strong Equivalence Principle (SEP). Therefore the SEP states that there is no experiment that we can make to distinguish between a local inertial frame which is free falling in a uniform gravitational field and an inertial frame which is in a region far from any gravitating masses. Vice versa we can simulate gravity by giving the reference frame an acceleration which will be indistinguishable from a uniform gravitational field[5],[6]. The gravitational force affects all objects in the same way and this is a characteristic only of the gravitational field. In fact if we consider for example the electromagnetic force we get

$$\overrightarrow{a} = \frac{q}{m_i}(\overrightarrow{E} + \overrightarrow{v} \wedge \overrightarrow{B}) \tag{2}$$

and different objects will experience different accelerations. In conclusion we can say that the theory of uniform gravitational field is a general relativity theory. Indeed the effects due to an uniform gravitational field are physically indistinguishable from the effects due to uniform acceleration. In this context, the acceleration is a relative physical quantity and we obtain a perfect generalization of the special relativity principle. Instead, it is well known that gravitational fields are not uniform and so it is crucial to ask whether we can extend the principle of relativity to accelerated reference frames. Can acceleration be made relative? Quite often the answer seems affirmative, and it is based on the general covariance principle. It is well know that general covariance is the invariance of the form of physical laws under arbitrary differentiable coordinate transformations and in many textbooks this principle is described as a general relativity principle. It is frequently written that the reason to call the Einstein gravitational theory "general relativity" is the fact that this theory generalizes the special relativity principle and this generalization is realized in principle of general covariance through which all frames of reference are equivalent. In our opinion it is crucial immediately to present the Kretschmann reflection that the general covariance does not make any assertions about the physical content of the physical laws, but only about their mathematical formulation, and Einstein entirely concurred with his view [7],[8],[9][10],[11],[12],[13],[14],[15]. We can give a generally covariant formulation for the Newtonian mechanics and the special relativity too and this clearly shows that the notion of general covariance is completely independent of relativity. Simple examples shows us how a non-covariant equation can be transformed into another equivalent and covariant [4]. This is the point that we believe is not sufficiently emphasized. Moreover there are generally covariant gravitational theories that at any point are reduced to classical mechanics. Pauli pointed out *"The generally covariant formulation of the physical laws acquires a physical content only through the principle of equivalence".* Indeed the foundation of Einstein theory is the following alternative version of the principle of equivalence:

A physical law is true if:

1) it is true in the absence of gravity, i.e. it reduces to the laws of special relativity when $g_{\mu\nu} \rightarrow \eta_{\mu\nu}$ and the Christoffel symbols vanish

2) In order to preserve their form under an arbitrary coordinate transformation, all equations must be generally covariant. This means that all equations must be expressed in a tensor form.

It is instructive to show that this formulation of general covariance follows from the principle of equivalence but it should be stressed that it does not in any way imply the physical equivalence of all reference frame similarly to what happens in special relativity. The previous two statements describe only the effects of gravitation.

3 Absolute or Relative Acceleration?

In Minkowski spacetime there is a difference between straight trajectories with no force present ($\frac{d^2x}{dt^2} = 0$) and curved trajectories ($\frac{d^2x}{dt^2} \neq 0$). In Einstein gravitational theory there is a difference between straight trajectories with no force present ($\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0$) and curved trajectories ($\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} \neq 0$). So, in Einstein gravitation, the key point is that inertial reference frames include objects at rest, in constant motion, or gravitationally accelerating. Therefore whether or not you are gravitationally accelerating depends on your frame of reference. Instead this is not true for other types of acceleration and non-gravitational acceleration is still absolute and it is independent of frame of reference. For a given gravitational field, vectorially subtracting the gravitational acceleration at the origin of a free falling reference frame from the gravitational acceleration at a given point, we obtain the tidal acceleration at a point with respect to the origin. If we consider the particle m at the point $(0, 0, z)$, we have

$$a_{tidal} = -\frac{GM}{(r+z)^2} + \frac{GM}{r^2} \tag{3}$$

for small values of z we get

$$a_{tidal} \cong 2z\frac{GM}{r^3} \tag{4}$$

and

$$F_{tidal} \cong 2z\frac{GMm}{r^3} \tag{5}$$

At the point (x, y, z) we have

$$F_{tidal} = -\frac{xGMm}{r^3}\widehat{i} - \frac{yGMm}{r^3}\widehat{j} + \frac{2zGMm}{r^3}\widehat{k} \tag{6}$$

If we have a rod in free fall, the tidal force generates a torque that produces an angular acceleration. The torque around the x axis is

$$M_x = \frac{3GM}{r^3}I_{yz} \tag{7}$$

where I_{yz} is the component $y - z$ of the product of inertia. For the x-component of angular momentum we have

$$\frac{dL_x}{dt} = \frac{3GM}{r^3}I_{yz} \tag{8}$$

If we consider the length $l \to 0$, both the torque and the angular momentum tends to zero but this is not true for the angular acceleration. Therefore we can use the angular acceleration to determine, even locally, the presence of a gravitational field. There are many ways to measure in an infinitesimal neighborhood the tidal field. For details you can read [4] In conclusion we notice the presence of a gravitational field, but we can not calculate any absolute gravitational acceleration. The equality of gravitational and inertial mass has as consequence only the physical indistinguishability (relativity principle) between an a constant accelerated frame of reference and a uniform gravitational field. Perhaps the confusion stems from the fact that Lorentz covariance correspond to the notions of physical equivalence and relativity. The covariance of Maxwell equations does not imply that the speed of light is the same in each reference but it is the invariance obtained by Lorentz covariance that leads to the principle of relativity.

4 Conclusion

In this brief paper we have stressed that the introductory textbooks on GR do not deepen the principle of general covariance and do not express clearly the difference between invariance and covariance. Lorentz invariance is a relativity principle but general covariance is not a general relativity principle. GR is not a fully relativistic theory but a geometric theory of gravitation built thanks to the principle of equivalence. Despite its name, the theory has failed in its original purpose and that is to create a totally relativistic physics. As long as the Mach principle will not be incorporated into GR, we are forced to admit that Newton was right. The acceleration (non-gravitational) is absolute.

References

[1] J. Barbour, H. Pfister, "Mach's Principle: From Newton's Bucket to Quantum Gravity", Birkhäuser, 1995

[2] E. Benedetto "Brief history of a relativistic century". Int. J. Theor. Phys. 50(8) (2011)

[3] A. Koyrè "Newtonian Studies" Harvard University Press, 1965

[4] H. Ohanian, R. Ruffini "Gravitation and Spacetime" W.W. Norton, New York (1994)

[5] C.M. Will "Theory and Experiment in Gravitational Physics" Cambridge University press 1993

[6] S. Kopeikin "Equivalence principle in cosmology" Proceedings of the Sixth Meeting on CPT and Lorentz Symmetry 2013

[7] I. Licata "Osservando la Sfinge". Di Renzo, Rome (2003)

[8] S. Weinberg "Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity" John Wiley & Sons 1972

[9] J.D. Norton "General covariance and the foundations of general relativity: eight decades of dispute" Rep. bog. Phys. 56 791-458 1993

[10] J.L. Anderson " Principles of Relativity Physics" Academic Press New York 1967

[11] V. Fock " The theory of Space, Time and Gravitation" Pergamon, Oxford 1964

[12] W. Pauli "Theory of Relativity" Dover Publications, 1981

[13] A. Einstein "The meaning of relativity". Princeton: Princeton University Press 1956

[14] E. Kretschmann "Über den physikalischen Sinn der Relativitätspostulate. A. Einsteins neue und seine ursprüngliche Relativitätstheorie". Annalen der Physik, 53, 575-614. 1917

[15] M. Friedman, Michael "Foundations of Space-Time Theories", Princeton University Press, 1983, 204-215

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published a paper titled Some General Remarks on the Relativity Principle. in which he tried to derive the most general theory consistent with the Relativity principle. What he came up with was a theory that resembled Einsteinâ€™s, but where Einstein used a value [math]\gamma = 1/(1-v^2/c^2)[/math] , Ignatowski uses a value [math]\gamma = 1/(1-\alpha v^2)[/math] . He didnâ€™t calculate it, but the velocity composition rule under Ignatowskiâ€™s formulation becomes [math]v_bc = (v_{ba}+v_{ac})/(1 + \alpha v_{ba}v_(bc})[/math] . General relativity incorporates a number of basic principles that correlate space-. time structure with physical objects and processes. Among them is the. Geodesic Principle: Free massive point particles traverse timelike. geodesics. One can think of it as a relativistic version of Newtonâ€™s first law of motion. It is often claimed that the geodesic principle can be recovered as a theorem in general relativity.Â In some cases, very specific assumptions are made about the constitution of the bodies in the sequence. A theorem in Thomas [6] and Taub [5] is of this type. There one takes each body to be a blob of perfect fluid, with everywhere non-negative isotropic pressure, that satisfies a strong constraint. In physics, the principle of relativity is the requirement that the equations describing the laws of physics have the same form in all admissible frames of reference. For example, in the framework of special relativity the Maxwell equations have the same form in all inertial frames of reference. In the framework of general relativity the Maxwell equations or the Einstein field equations have the same form in arbitrary frames of reference. The relativity principle does not depend on transformations but instead, transformations can be developed in order to match the PoR.Â * That the aether of general relativity emerges from Lorentz's aether by "relativization", that is, removing also the last mechanical property - its "state of rest". * But this aether may not be thought of as endowed with the quality characteristic of ponderable media, as consisting of parts which may be tracked through time. The idea of motion may not be applied to it. Thus the very "idea of motion" of the aether is completely banned in Einstein's theory (both special and general), but remains as a hidden entity in Lorentz's theory - this is what brings people to distinguish between "special relativity" and "Lorentz ether theory". harrylin said Nevertheless, general relativity assumes each small region of space and time asymptotically approaches â€œflatnessâ€, and in these infinitesimal regions the metrical properties are those of special relativity, which weâ€™ve already seen are based epistemologically on the principle of inertia.Â Hence â€œgeneral covarianceâ€ can serve as a guide only if combined with some other concept, such as the notion that physical laws ought to have particularly simple representations when expressed in generally covariant form.Â Einsteinâ€™s claim that general relativity dispenses with the principle of inertia might seem to be supported by the fact that, uniquely among field theories, the equations of motion in general relativity need not be postulated separately from the field equations.