MODELING AND ANALYSIS OF COMPLEX PRODUCTION SYSTEMS

Jingshan Li
Manufacturing Systems Research Laboratory
General Motors Research & Development Center
Warren, MI 48090-9055, USA
Email: jingshan.li@gm.com

Abstract: *In this paper, a system-theoretic method, referred to as overlapping decomposition, is presented to estimate the production rate of a complex production system with assembly, parallel, rework, feed-forward and scrap operations. The idea of the method is to decompose the system into overlapped serial lines, and to modify the parameters of overlapping machines to accommodate the effects of other lines. Convergent iteration procedures are introduced to estimate system performance and result in a high accuracy.*

Keywords: *Complex production system, overlapping decomposition, production rate.*

1 INTRODUCTION

Fast and accurate analysis of system production rate (PR) is important for design, operation and continuous improvement of production systems. Analytical results have been developed for serial lines and assembly lines. However, for systems with other complex structures, such as parallel, rework, feed-forward, etc., which are widely encountered in large volume manufacturing industry, the study is quite limited.

Parallel lines are often used to increase production capacity or product varieties in many production systems. In parallel systems, a main line is split into one or more parallel lines, and finally merge into a main line again. In some systems, some parts may bypass some operations or are routed to a specific line and merged into the main line later. This constructs the feed-forward (or specific routing) lines. In addition, in many production systems, one or more rework loops are attached to the main line to reprocess or multi-process the jobs in order to attain higher quality standards and produce multiple types of parts. For some parts with severe defects, they maybe scraped from the line. Moreover, in assembly systems, two or more parts are brought together to form a single part. Part of or all above operations comprise a complex production system.

Except for some results in assembly systems (see representative papers [1]-[5] and their references), the study on other complex systems is limited. References [6]-[12] study the series-parallel lines with each stage consisting of (single) parallel machines. However, most parallel systems have multiple machines in each parallel line, and the current literature does not provide an effective method to study its performance. No related work on feed-forward operations is discovered in the literature. Although there is limited study on closed loop production systems (see papers [13]-[15] on closed loop serial lines with constant number of carriers), the rework loop problem is not addressed directly. Moreover, no analytical methods are found to study the systems with different complex operations mixed together. The main contribution of this paper is to present a system-theoretic method, referred to as *overlapping decomposition*, to analyze such systems, for instance, systems with assembly, parallel, rework, feed-forward and scrap operations.

The remainder of this paper is structured as follows: Section 2 formulates the problem. The idea of overlapping decomposition is introduced in Section 3. Section 4 presents the recursive procedures to estimate system production rate. The conclusions are formulated in Section 5. Due to page limitation, all proofs are omitted and can be found in [16].

2 PROBLEM FORMULATION

A typical structure of complex production system is shown in Figure 1, where the circles represent machines and rectangles are buffers. The system consists of one main line, one feeder line

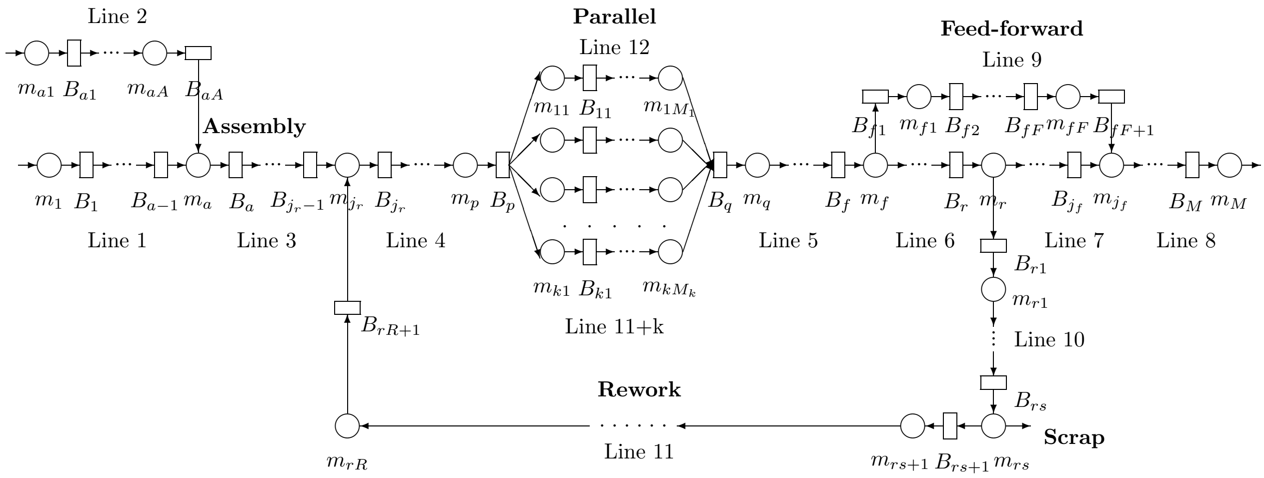

Figure 1: A typical structure of complex production system

(assembly), k parallel lines, one feed-forward line and one rework loop. Subscripts a, f, r are used to denote the machines or buffers in feeder line, feed-forward line and rework loop, respectively. Lines 1 to $11+k$ are defined in Subsection 3.1. A description of the notations of machines and buffers is introduced below:

Main line:	$m_1, \ldots, m_p, m_q, \ldots, m_M, \quad B_1, \ldots, B_p, B_q, \ldots, B_{M-1};$
Feeder line:	$m_{a1}, \ldots, m_{aA}, \quad B_{a1}, \ldots, B_{aA};$
Feed-forward line:	$m_{f1}, \ldots, m_{fF}, \quad B_{f1}, \ldots, B_{fF+1};$
Rework loop:	$m_{r1}, \ldots, m_{rR}, \quad B_{r1}, \ldots, B_{rR+1};$
Parallel lines:	$m_{ij}, i = 1, \ldots, k, j = 1, \ldots, M_i, \quad B_{ij}, i = 1, \ldots, k, j = 1, \ldots, M_i - 1;$
Merge machines:	m_a (assembly merge), m_{j_r} (rework merge), m_{j_f} (feed-forward merge);
Split machines:	m_r (rework split), m_f (feed-forward split), m_{rs} (scrap);
Split/merge buffers :	B_p (parallel split), B_q (parallel merge).

To study the system performance, the following assumptions are introduced:

(i) Each machine has two states: up and down. When up, for machines in main line, feeder line, feed-forward line and rework loop, each of them is capable of producing with the rate S parts per unit of time. For machines in parallel lines, each machine, m_{ij}, $i = 1, \ldots, k$, $j = 1, \ldots, M_i$, in parallel line i, can produce with the rate S_i parts per unit of time when the machine is up. Moreover, $\sum_{i=1}^{k} S_i = S$. When the machine is down, no production takes place.

(ii) The uptime and the downtime of each machine m_i, $\forall i$, are random variables distributed exponentially with parameters p_i and r_i, respectively.

(iii) Each buffer B_i, $\forall i$, has a finite capacity N_i, $0 \le N_i < \infty$.

(iv) A machine is starved at time t if upstream buffer is empty at time t. Machines m_1 and m_{a1} are never starved. In addition, machine m_a is starved if either buffer, B_{a-1} or B_{aA}, is empty. It is assumed that machine m_{j_r} always takes part from B_{rR+1} (rework loop) first if it is not empty. Moreover, Machine m_{j_f} can take parts either from B_{j_f} or B_{fF+1} without any priority.

(v) A machine is blocked at time t if downstream buffer is full at time t. Machine m_M is never blocked. Machine m_r is blocked by main line if it produces a good part and buffer B_{r+1} is full, and blocked by rework loop if a defective part is produced and B_{r1} is full. Analogously, machine m_f can be blocked by main line and feed-forward line correspondingly.

(vi) A part is defective with probability α at machine m_r, $0 \leq \alpha < 1$. At machine m_f, a part has probability β, $0 \leq \beta < 1$ to be sent to feed-forward line. A severe defective part is scraped with probability γ at machine m_s, $0 \leq \gamma < 1$. Probabilities α, β and γ are referred to as the rework, feed-forward and scrap rates, respectively.

(vii) The first machine in each parallel line, machine m_{i1} , $i = 1, \ldots, k$, has equal probability of taking a part from buffer B_p if it is not blocked, and the last machine in each parallel line, machine m_{iM_i} has equal probability of sending a part to buffer B_q if it is not starved.

The problem addressed is as follows: *Given production system (i)-(vii), develop a method for evaluating the production rate as a function of the system parameters.* A solution to the problem is given in Section 4.

3 IDEA OF THE APPROACH

3.1 Decomposition

Due to complexity of the system, exact analysis is intractable. Therefore, an approximation method is needed. The idea of the approximation is to decompose the complex production system into a set of serial lines, with the first or last machines in one serial line overlapping with the first or last machines in another serial line, and to modify the overlapping machines appropriately to accommodate the effects of machines and buffers in other lines.

System (i)-(vii) shown in Figure 1 is decomposed into $11+k$ serial lines (where k is the number of parallel lines). A description of the decomposed lines is shown below:

$$
\begin{aligned}
&\text{Line 1:} && m_1, m_2, \ldots, m_{a-1}, m_a, \quad B_1, \ldots, B_{a-1};\\
&\text{Line 2:} && m_{a1}, m_{a2}, \ldots, m_{aA}, m_a, \quad B_{a1}, \ldots, B_{aA};\\
&\text{Line 3:} && m_a, m_{a+1}, \ldots, m_{j_r-1}, m_{j_r}, \quad B_a, \ldots, B_{j_r-1};\\
&\text{Line 4:} && m_{j_r}, m_{j_r+1}, \ldots, m_p, m_s, \quad B_{j_r}, \ldots, B_p;\\
&\text{Line 5:} && m_m, m_q, \ldots, m_{f-1}, m_f, \quad B_q, \ldots, B_f;\\
&\text{Line 6:} && m_f, m_{f+1}, \ldots, m_{r-1}, m_r, \quad B_{f+1}, \ldots, B_r;\\
&\text{Line 7:} && m_r, m_{r+1}, \ldots, m_{j_f-1}, m_{j_f}, \quad B_{r+1}, \ldots, B_{j_f};\\
&\text{Line 8:} && m_{j_f}, m_{j_f+1}, \ldots, m_{M-1}, m_M, \quad B_{j_f+1}, \ldots, B_M;\\
&\text{Line 9:} && m_f, m_{f1}, \ldots, m_{fF}, m_{j_f}, \quad B_{f1}, \ldots, B_{fF+1};\\
&\text{Line 10:} && m_r, m_{r1}, \ldots, m_{rs-1}, m_{rs}, \quad B_{r1}, \ldots, B_{rs};\\
&\text{Line 11:} && m_{rs}, m_{rs+1}, \ldots, m_{rR}, m_{j_r}, \quad B_{rs+1}, \ldots, B_{rR+1};\\
&\text{Line } 11+i\text{:} && m_{i1}, \ldots, m_{iM_i}, \quad B_{i1}, \ldots, B_{iM_i-1}, \qquad i = 1, \ldots, k,
\end{aligned}
$$

where machines m_s and m_m are the aggregated parallel split and merge machines, respectively. A formal definition of m_s and m_m is introduced in Subsection 4.2.

3.2 Modification of Split and Merge Machines

To modify the overlapping (i.e., split and merge) machines, a convergent procedure to evaluate the production rate of serial lines is needed. Such procedures have been developed in [16] and [17]. In order to use these procedures, the first and last machines in a serial line should not be starved or blocked. Therefore, we introduce fictitious machines by assuming that the parameters of the first or last machines in one line are modified so as to account for the existence of other machines and buffers in other lines. In the following, detailed adjustments are illustrated.

3.2.1 Assembly merge

First, assembly machine m_a in line 3 (Figure 1) is adjusted by assuming that it is not starved by either main line or feeder line, which implies that

$$r_a^3 = r_a \text{Prob}\{m_a \text{ is not starved by } B_{a-1}\}\text{Prob}\{m_a \text{ is not starved by } B_{aA}\}.$$

Here and in subsequent discussions, superscript j, $j = 1, \ldots, 11+k$, is used to denote the overlapping machine in line j. r_a^3 is introduced by considering the starving time as m_a's downtime (from the point of view of the machines downstream of m_a). Therefore, fictitious machine m_a^3's average downtime $\frac{1}{r_a^3}$ can be equivalent to $\frac{1}{r_a \text{Prob}\{m_a \text{ is not starved by either } B_{a-1} \text{ or } B_{aA}\}}$. p_a^3 is selected by following the conservation of flow such that

$$\frac{r_a^3}{r_a^3 + p_a^3} = \frac{r_a}{r_a + p_a}\text{Prob}\{m_a \text{ is not starved by either } B_{a-1} \text{ or } B_{aA}\}.$$

It follows that

$$p_a^3 = p_a + r_a - r_a^3.$$

Similar adjustments are used in all subsequent discussions when modifying the first and last machines. Then, for m_a in lines 1 and 2, the blocking time can also be considered as m_a's downtime from the point of view of the machines upstream of m_a. Thus, we obtain,

$$\begin{aligned} r_a^1 &= r_a \text{Prob}\{m_a \text{ is not starved by } B_{aA}\}\text{Prob}\{m_a \text{ is not blocked by } B_a\}, \\ r_a^2 &= r_a \text{Prob}\{m_a \text{ is not starved by } B_{a-1}\}\text{Prob}\{m_a \text{ is not blocked by } B_a\}. \end{aligned}$$

3.2.2 Parallel split and merge

For each parallel line i, $i = 1, \ldots, k$, we introduce the following modifications:

$$\begin{aligned} r_{i1}^j &= r_{i1}\text{Prob}\{m_{i1} \text{ is not starved by } B_p\}, \\ r_{iM_i}^j &= r_{iM_i}\text{Prob}\{m_{iM_i} \text{ is not blocked by } B_q\}, \quad j = 11+i. \end{aligned}$$

For lines 4 and 5, first we define machines m_{i1}^4 and $m_{iM_i}^5$ as

$$\begin{aligned} r_{i1}^4 &= r_{i1}\text{Prob}\{m_{i1} \text{ is not blocked by } B_{i1}\}, \\ r_{iM_i}^5 &= r_{iM_i}\text{Prob}\{m_{iM_i} \text{ is not starved by } B_{iM_i-1}\}. \end{aligned}$$

Then machines m_{i1}^4 and $m_{iM_i}^5$, $i = 1, \ldots, k$, are aggregated into machines m_s and m_m, respectively.

3.2.3 Rework split and merge

For the rework split machine m_r in lines 6, 7 and 10, we obtain

$$\begin{aligned} r_r^6 &= r_r[1 - \alpha\text{Prob}\{m_r \text{ is blocked by } B_{r1}\} - (1-\alpha)\text{Prob}\{m_r \text{ is blocked by } B_{r+1}\}], \\ r_r^7 &= r_r(1-\alpha)\text{Prob}\{m_r \text{ is not starved}\}, \\ r_r^{10} &= r_r\alpha\text{Prob}\{m_r \text{ is not starved}\}. \end{aligned}$$

For the rework merge machine m_{j_r} in lines 3, 4 and 11, we have

$$\begin{aligned} r_{j_r}^3 &= r_{j_r}\text{Prob}\{m_{j_r} \text{ is not blocked by } B_{j_r}\}\text{Prob}\{m_{j_r} \text{ is starved by } B_{rR+1}\}, \\ r_{j_r}^4 &= r_{j_r}\text{Prob}\{m_{j_r} \text{ is not starved by } B_{j_r-1} \text{ and } B_{rR+1}\}, \\ r_{j_r}^{11} &= r_{j_r}\text{Prob}\{m_{j_r} \text{ is not blocked by } B_{j_r}\}. \end{aligned}$$

3.2.4 Feed-forward split and merge

For feed-forward split machine m_f in lines 5, 6 and 9, let

$$\begin{aligned} r_f^5 &= r_f[1-\beta\text{Prob}\{m_f \text{ is blocked by } B_{f1}\} - (1-\beta)\text{Prob}\{m_f \text{ is blocked by } B_{f+1}\}], \\ r_f^6 &= r_f(1-\beta)\text{Prob}\{m_f \text{ is not starved}\}, \\ r_f^9 &= r_f\beta\text{Prob}\{m_f \text{ is not starved}\}. \end{aligned}$$

For feed-forward merge machine m_{j_f} in lines 7 - 9, let

$$\begin{aligned} r_{j_f}^7 &= r_{j_f}(1-\beta)\text{Prob}\{m_{j_f} \text{ is not blocked}\}, \\ r_{j_f}^8 &= r_{j_f}\text{Prob}(1-\text{Prob}\{m_{j_f} \text{ is starved by } B_{fF+1}\}\text{Prob}\{m_{j_f} \text{ is starved by } B_{j_f}\}), \\ r_{j_f}^9 &= r_{j_f}\beta\text{Prob}\{m_{j_f} \text{ is not blocked}\}. \end{aligned}$$

3.2.5 Scrap

For scrap machine m_{rs} in lines 10 and 11, we have

$$\begin{aligned} r_{rs}^{10} &= r_{rs}(1-\gamma-(1-\gamma)\text{Prob}\{m_{rs} \text{ is blocked}\}), \\ r_{rs}^{11} &= r_{rs}(1-\gamma)\text{Prob}\{m_{rs} \text{ is not starved}\}. \end{aligned}$$

3.3 Iterations

If we know the above starving and blocking probabilities, using the serial line evaluation method, we can calculate the production rates of all lines and analyze the performance of the whole system. Since these probabilities are unknown, we introduce iterations. We begin with line 1 (Figure 1). Assume we know the probability that m_a is blocked and the probability that m_a is starved by feeder line, modify m_a to m_a^1 according to Subsection 3.2. Use the serial line analysis procedure, we calculate the probability that m_a is starved by main line. (We denote the calculations of blocking and starving probabilities as operators Φ_b and Φ_s, respectively, and define them in Subsection 4.1.) Use this probability, we proceed to line 2 and calculate the probability that m_a is starved by feeder line. Continue this process to lines 3 to $11+k$, calculate all the starving and blocking probabilities, respectively. Then, use them for the second iteration, alternating among $11+k$ lines. It is shown below that the procedure is convergent and the system production rate equals to the production rate of line 8. A formal description of the iteration procedure is introduced next.

4 RECURSIVE PROCEDURES

4.1 Operators Φ_b and Φ_s

Consider a serial production line of machines m_1 to m_M and buffer B_1 to B_{M-1}, which can be viewed as a special case of system (i)-(v) with only a main line. Introduce operators Φ_b and Φ_s which are used to calculate the probabilities that the first machine is blocked and the last machine is starved in a serial line, respectively. A definition of operators Φ_b and Φ_s is introduced below:

$$\Phi_b = 1 - \frac{r_1^b}{p_1^b + r_1^b} \Big/ \frac{r_1}{p_1 + r_1}, \qquad \Phi_s = 1 - \frac{r_M^f}{p_M^f + r_M^f} \Big/ \frac{r_M}{p_M + r_M}, \tag{1}$$

where p_1^b, r_1^b, p_M^f and r_M^f are defined through the following procedure introduced in [16]:

Procedure 1

$$\begin{aligned} r_i^b(l+1) &= r_i - r_i Q\Big(p_{i+1}^b(l+1), r_{i+1}^b(l+1), p_i^f(l), r_i^f(l), N_i\Big), \\ p_i^b(l+1) &= p_i + r_i - r_i^b(l+1), \qquad\qquad i = 1, \cdots, M-1, \\ r_i^f(l+1) &= r_i - r_i Q\Big(p_{i-1}^f(l+1), r_{i-1}^f(l+1), p_i^b(l+1), r_i^b(l+1), N_{i-1}\Big), \\ p_i^f(l+1) &= p_i + r_i - r_i^f(l+1), \qquad\qquad i = 2, \cdots, M, \end{aligned} \tag{2}$$

with boundary conditions

$$p_1^f(l) = p_1, \quad r_1^f(l) = r_1, \quad p_M^b(l) = p_M, \quad r_M^b(l) = r_M, \qquad l = 0, 1, 2, \ldots,$$

and initial conditions

$$p_i^f(0) = p_i, \quad r_i^f(0) = r_i, \qquad i = 2, \cdots, M-1.$$

Here functions $Q(\cdot)$ and ϕ (see [17]) are defined by

$$Q(p_1, r_1, p_2, r_2, N) = \begin{cases} \frac{(1-e_1)(1-\phi)}{1-\phi e^{-\beta N}}, & if \quad \frac{p_1}{r_1} \neq \frac{p_2}{r_2}, \\ \frac{p_1(p_1+p_2)(r_1+r_2)}{(p_1+r_1)[(p_1+p_2)(r_1+r_2)+p_2 r_1(p_1+p_2+r_1+r_2)N]}, & if \quad \frac{p_1}{r_1} = \frac{p_2}{r_2}, \end{cases} \tag{3}$$

$$\phi = \frac{e_1(1-e_2)}{e_2(1-e_1)}, \qquad e_i = \frac{r_i}{p_i + r_i}, \qquad i = 1, 2. \tag{4}$$

It is shown in [16] that procedure 1 is convergent and has a unique solution, i.e., p_M^f, r_M^f, p_1^b and r_1^b exist. Let $PR(\cdot)$ denote the calculation of production rate through procedure 1. Then the system production rate can be estimated as

$$\widehat{PR} = PR(p_1, r_1, \ldots, p_M, r_M, S, N_1, \ldots, N_{M-1}) = \frac{S r_M^f}{p_M^f + r_M^f}. \tag{5}$$

With operators Φ_b and Φ_s, we introduce the iteration procedure to analyze system (i)-(vii) in Subsection 4.3.

4.2 Aggregation of Multiple Parallel Machines

To adopt the approach described in Section 3.2, an aggregation of multiple parallel machines into one single machine is needed. Consider the following multiple machine system with machines m_{11}, …, m_{k1} in parallel (Figure 2), where machines m_{i1}, $i = 1$, …, k, are described by assumption (i) and (ii). To aggregate them into one equivalent single machine m_s, we assume the uptime

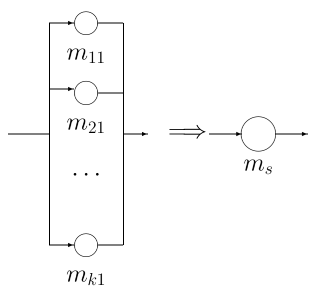

Figure 2: Aggregation of parallel machines into one single machine

and the downtime of the aggregated machine m_s are random variables distributed exponentially with parameters p_s and r_s, respectively. Then, p_s and r_s can be calculated as follows (see [16] for details):

Proposition 1 *The aggregated machine m_s is described by the following parameters:*

$$p_s = S\frac{p_{11}r_{11}\prod_{i=2}^{k}(p_{i1}+r_{i1})}{\sum_{i=1}^{k}S_i r_{i1}\prod_{j=1,j\neq i}^{k}(p_{j1}+r_{j1})}, \qquad r_s = S\frac{p_{11}r_{11}\prod_{i=2}^{k}(p_{i1}+r_{i1})}{\sum_{i=1}^{k}S_i p_{i1}\prod_{j=1,j\neq i}^{k}(p_{j1}+r_{j1})}. \tag{6}$$

where $S = \sum_{i=1}^{k} S_i$, and moreover,

$$S\frac{r_s}{p_s+r_s} = \sum_{i=1}^{k} S_i \frac{r_{i1}}{p_{i1}+r_{i1}}. \tag{7}$$

4.3 Recursive Procedure

For simplification, introduce the following notations:

$$\begin{aligned}
s_x &= \text{Prob}\{\text{machine } m_x \text{ is starved}\}, \quad x = f, iM_i, r, rs \text{ and } s,\\
b_y &= \text{Prob}\{\text{machine } m_y \text{ is blocked}\}, \quad y = a, i1, j_f, j_r, m \text{ and } rs,\\
s_{xy} &= \text{Prob}\{\text{machine } m_x (x = a, j_f, j_r) \text{ is starved by main line } (y = m)\text{, feeder line } (y = a),\\
&\qquad \text{feed-forward line } (y = f) \text{ and rework loop } (y = r)\text{, respectively}\},\\
b_{xy} &= \text{Prob}\{\text{machine } m_x (x = f, r) \text{ is blocked by main line } (y = m)\text{, feed-forward line } (y = f)\\
&\qquad \text{and rework loop } (y = r)\text{, respectively}\}.
\end{aligned}$$

Formally, the recursive procedure is:

Procedure 2

$$\begin{aligned}
\underline{\text{Line 1}}:\quad r_a^1(n+1) &= r_a[1-b_a(n)][1-s_{aa}(n)],\\
p_a^1(n+1) &= p_a + r_a - r_a^1(n+1), \qquad\qquad (8)\\
s_{am}(n+1) &= \Phi_s\Big(p_1, r_1, \dots, p_a^1(n+1), r_a^1(n+1), N_1, \dots, N_{a-1}\Big),
\end{aligned}$$

$$\begin{aligned}
\underline{\text{Line 2}}:\quad r_a^2(n+1) &= r_a[1-b_a(n)][1-s_{am}(n+1)],\\
p_a^2(n+1) &= p_a + r_a - r_a^2(n+1), \qquad\qquad (9)\\
s_{aa}(n+1) &= \Phi_s\Big(p_{a1}, r_{a1}, \dots, p_a^2(n+1), r_a^2(n+1), N_{a1}, \dots, N_{aA}\Big),
\end{aligned}$$

$$\begin{aligned}
\underline{\text{Line 3}}:\quad r_a^3(n+1) &= r_a[1-s_{am}(n+1)][1-s_{aa}(n+1)],\\
p_a^3(n+1) &= p_a + r_a - r_a^3(n+1),\\
r_{j_r}^3(n+1) &= r_{j_r} s_{j_r r}(n)[1-b_{j_r}(n)],\\
p_{j_r}^3(n+1) &= p_{j_r} + r_{j_r} - r_{j_r}^3(n+1), \qquad\qquad (10)\\
s_{j_r m}(n+1) &= \Phi_s\Big(p_a^3(n+1), r_a^3(n+1), \dots, p_{j_r}^3(n+1), r_{j_r}^3(n+1), N_a, \dots, N_{j_r-1}\Big),\\
b_a(n+1) &= \Phi_b\Big(p_a^3(n+1), r_a^3(n+1), \dots, p_{j_r}^3(n+1), r_{j_r}^3(n+1), N_a, \dots, N_{j_r-1}\Big),
\end{aligned}$$

$$\begin{aligned}
\underline{\text{Line 4}}:\quad r_{j_r}^4(n+1) &= r_{j_r}[1-s_{j_r m}(n+1)s_{j_r r}(n)],\\
p_{j_r}^4(n+1) &= p_{j_r} + r_{j_r} - r_{j_r}^4(n+1),\\
r_{i1}^4(n+1) &= r_{i1}[1-b_{i1}(n)], \quad i = 1, \dots, k,\\
p_{i1}^4(n+1) &= p_{i1} + r_{i1} - r_{i1}^4(n+1), \quad i = 1, \dots, k,\\
p_s(n+1) &= S\frac{p_{11}^4(n+1)r_{11}^4(n+1)\prod_{i=2}^{k}(p_{i1}^4(n+1)+r_{i1}^4(n+1))}{\sum_{i=1}^{k}S_i r_{i1}^4(n+1)\prod_{j=1,j\neq i}^{k}(p_{j1}^4(n+1)+r_{j1}^4(n+1))}, \qquad\qquad (11)
\end{aligned}$$

$$
\begin{aligned}
r_s(n+1) &= S\frac{p_{11}^4(n+1)r_{11}^4(n+1)\prod_{i=2}^k(p_{i1}^4(n+1)+r_{i1}^4(n+1))}{\sum_{i=1}^k S_i p_{i1}^4(n+1)\prod_{j=1,j\neq i}^k(p_{j1}^4(n+1)+r_{j1}^4(n+1))},\\
b_{j_r}(n+1) &= \Phi_b\Big(p_{j_r}^4(n+1), r_{j_r}^4(n+1), \ldots, p_s(n+1), r_s(n+1), N_{j_r}, \ldots, N_p\Big),\\
s_s(n+1) &= \Phi_s\Big(p_{j_r}^4(n+1), r_{j_r}^4(n+1), \ldots, p_s(n+1), r_s(n+1), N_{j_r}, \ldots, N_p\Big),
\end{aligned}
$$

<u>Line 5</u>:

$$
\begin{aligned}
r_f^5(n+1) &= r_f[1-\beta b_{ff}(n)-(1-\beta)b_{fm}(n)],\\
p_f^5(n+1) &= p_f + r_f - r_f^5(n+1),\\
r_{iM_i}^5(n+1) &= r_{iM_i}[1-s_{iM_i}(n)], \qquad i=1,\ldots,k,\\
p_{iM_i}^5(n+1) &= p_{iM_i} + r_{iM_i} - r_{iM_i}^5(n+1), \qquad i=1,\ldots,k,\\
p_m(n+1) &= S\frac{p_{1M_1}^5(n+1)r_{1M_1}^5(n+1)\prod_{i=2}^k(p_{iM_i}^5(n+1)+r_{iM_i}^5(n+1))}{\sum_{i=1}^k S_i r_{iM_i}^5(n+1)\prod_{j=1,j\neq i}^k(p_{jM_j}^5(n+1)+r_{jM_j}^5(n+1))}, && (12)\\
r_m(n+1) &= S\frac{p_{1M_1}^5(n+1)r_{1M_1}^5(n+1)\prod_{i=2}^k(p_{iM_i}^5(n+1)+r_{iM_i}^5(n+1))}{\sum_{i=1}^k S_i p_{iM_i}^5(n+1)\prod_{j=1,j\neq i}^k(p_{jM_j}^5(n+1)+r_{jM_j}^5(n+1))},\\
s_f(n+1) &= \Phi_s\Big(p_m(n+1), r_m(n+1), \ldots, p_f^5(n+1), r_f^5(n+1), N_q, \ldots, N_f\Big),\\
b_m(n+1) &= \Phi_b\Big(p_m(n+1), r_m(n+1), \ldots, p_f^5(n+1), r_f^5(n+1), N_q, \ldots, N_f\Big),
\end{aligned}
$$

<u>Line 6</u>:

$$
\begin{aligned}
r_f^6(n+1) &= r_f(1-\beta)[1-s_f(n+1)],\\
p_f^6(n+1) &= p_f + r_f - r_f^6(n+1),\\
r_r^6(n+1) &= r_r[1-\alpha b_{rr}(n)-(1-\alpha)b_{rm}(n)],\\
p_r^6(n+1) &= p_r + r_r - r_r^6(n+1), && (13)\\
b_{fm}(n+1) &= \Phi_b\Big(p_f^6(n+1), r_f^6(n+1), \ldots, p_r^6(n+1), r_r^6(n+1), N_{f+1}, \ldots, N_r\Big),\\
s_r(n+1) &= \Phi_s\Big(p_f^6(n+1), r_f^6(n+1), \ldots, p_r^6(n+1), r_r^6(n+1), N_{f+1}, \ldots, N_r\Big),
\end{aligned}
$$

<u>Line 7</u>:

$$
\begin{aligned}
r_r^7(n+1) &= r_r[1-s_r(n+1)](1-\alpha),\\
p_r^7(n+1) &= p_r + r_r - r_r^7(n+1),\\
r_{j_f}^7(n+1) &= r_{j_f}(1-\beta)[1-b_{j_f}(n)],\\
p_{j_f}^7(n+1) &= p_{j_f} + r_{j_f} - r_{j_f}^7(n+1), && (14)\\
b_{rm}(n+1) &= \Phi_b\Big(p_r^7(n+1), r_r^7(n+1), \ldots, p_{j_f}^7(n+1), r_{j_f}^7(n+1), N_{r+1}, \ldots, N_{j_f}\Big),\\
s_{j_f m}(n+1) &= \Phi_s\Big(p_r^7(n+1), r_r^7(n+1), \ldots, p_{j_f}^7(n+1), r_{j_f}^7(n+1), N_{r+1}, \ldots, N_{j_f}\Big),
\end{aligned}
$$

<u>Line 8</u>:

$$
\begin{aligned}
r_{j_f}^8(n+1) &= r_{j_f}[1-s_{j_f m}(n+1)s_{j_f f}(n)],\\
p_{j_f}^8(n+1) &= p_{j_f} + r_{j_f} - r_{j_f}^8(n+1), && (15)\\
b_{j_f}(n+1) &= \Phi_b\Big(p_{j_f}^8(n+1), r_{j_f}^8(n+1), \ldots, p_M, r_M, N_{j_f+1}, \ldots, N_M\Big),
\end{aligned}
$$

<u>Line 9</u>:

$$
\begin{aligned}
r_f^9(n+1) &= r_f\beta[1-s_f(n+1)],\\
p_f^9(n+1) &= p_f + r_f - r_f^9(n+1),\\
r_{j_f}^9(n+1) &= r_{j_f}\beta[1-b_{j_f}(n+1)],\\
p_{j_f}^9(n+1) &= p_{j_f} + r_{j_f} - r_{j_f}^9(n+1), && (16)\\
s_{j_f f}(n+1) &= \Phi_s\Big(p_f^9(n+1), r_f^9(n+1), \ldots, p_{j_f}^9(n+1), r_{j_f}^9(n+1), N_{f1}, \ldots, N_{fF+1}\Big),
\end{aligned}
$$

$$
\begin{aligned}
b_{ff}(n+1) &= \Phi_b\Big(p_f^9(n+1), r_f^9(n+1), \ldots, p_{j_f}^9(n+1), r_{j_f}^9(n+1), N_{f1}, \ldots, N_{fF+1}\Big),
\end{aligned}
$$

$$
\begin{aligned}
\underline{\text{Line 10}}: \quad r_r^{10}(n+1) &= r_r\alpha[1 - s_r(n+1)], \\
p_r^{10}(n+1) &= p_r + r_r - r_r^{10}(n+1), \\
r_{rs}^{10}(n+1) &= r_{rs}(1-\gamma)[1 - b_{rs}(n)], \\
p_{rs}^{10}(n+1) &= p_{rs} + r_{rs} - r_{rs}^{10}(n+1), \\
b_{rr}(n+1) &= \Phi_b\Big(p_r^{10}(n+1), r_r^{10}(n+1), \ldots, p_{rs}^{10}(n+1), r_{rs}^{10}(n+1), N_{r1}, \ldots, N_{rs}\Big), \\
s_{rs}(n+1) &= \Phi_s\Big(p_r^{10}(n+1), r_r^{10}(n+1), \ldots, p_{rs}^{10}(n+1), r_{rs}^{10}(n+1), N_{r1}, \ldots, N_{rs}\Big),
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
\underline{\text{Line 11}}: \quad r_{rs}^{11}(n+1) &= r_{rs}(1-\gamma)[1 - s_{rs}(n+1)], \\
p_{rs}^{11}(n+1) &= p_r + r_r - r_{rs}^{11}(n+1), \\
r_{j_r}^{11}(n+1) &= r_{j_r}[1 - b_{j_r}(n+1)], \\
p_{j_r}^{11}(n+1) &= p_{j_r} + r_{j_r} - r_{j_r}^{11}(n+1), \\
b_{rs}(n+1) &= \Phi_b\Big(p_{rs}^{11}(n+1), r_{rs}^{11}(n+1), \ldots, p_{j_r}^{11}(n+1), r_{j_r}^{11}(n+1), N_{rs+1}, \ldots, N_{rR+1}\Big), \\
s_{j_r r}(n+1) &= \Phi_s\Big(p_{rs}^{11}(n+1), r_{rs}^{11}(n+1), \ldots, p_{j_r}^{11}(n+1), r_{j_r}^{11}(n+1), N_{rs+1}, \ldots, N_{rR+1}\Big),
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
\underline{\text{Line } j}: \quad r_{i1}^{j}(n+1) &= r_{i1}[1 - s_s(n+1)], \qquad i = 1, \ldots, k, j = 11 + i, \\
p_{i1}^{j}(n+1) &= p_{i1} + r_{i1} - r_{i1}^{j}(n+1), \\
r_{iM_i}^{j}(n+1) &= r_{iM_i}[1 - b_m(n+1)], \\
p_{iM_i}^{j}(n+1) &= p_{iM_i} + r_{iM_i} - r_{iM_i}^{j}(n+1), \\
b_{i1}(n+1) &= \Phi_b\Big(p_{i1}^{j}(n+1), r_{i1}^{j}(n+1), \ldots, p_{iM_i}^{j}(n+1), r_{iM_i}^{j}(n+1), N_{i1}, \ldots, N_{iM_i-1}\Big), \\
s_{iM_i}(n+1) &= \Phi_s\Big(p_{i1}^{j}(n+1), r_{i1}^{j}(n+1), \ldots, p_{iM_i}^{j}(n+1), r_{iM_i}^{j}(n+1), N_{i1}, \ldots, N_{iM_i-1}\Big),
\end{aligned} \tag{19}
$$

$$
n = 0, 1, 2, \ldots,
$$

with initial conditions

$$
s_x(0) = 0, \qquad b_y(0) = 1, \qquad \forall x, y.
$$

4.4 Convergence and Accuracy

Theorem 1 *Under assumptions (i)-(vii), recursive procedure 2 is convergent and has a unique solution.*

Therefore, limits $p_{j_f}^8$ and $r_{j_f}^8$ exist. Using these limits, the production rate of system (i)-(vii) can be estimated as:

$$
\widehat{PR} = PR(p_{j_f}^8, r_{j_f}^8, \ldots, p_M, r_M, S, N_{j_f+1}, \ldots, N_M). \tag{20}
$$

The accuracy of method has been investigated numerically. It is shown in [16] that the estimation results in a high precision. Due to page limitation, this part is omitted.

5 CONCLUSIONS

In this paper, a system-theoretic approach, referred to as *overlapping decomposition*, is presented to model and analyze complex production systems. The approach is applicable to study systems with assembly, parallel, rework, feed-forward and scrap operations. The iteration procedure is shown to be convergent and result in high accuracy.

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References

[1] S.B. Gershwin, "Assembly/Disassembly Systems: An Efficient Decomposition Algorithm for Tree-Structured Networks", *IIE Trans.*, vol. 23, pp. 315-331, 1991.

[2] M. Di Mascolo, R. David and Y. Dallery, "Modeling and Analysis of Assembly Systems with Unreliable Machines and Finite Buffers", *IIE Trans.*, vol. 23, pp. 315-330, 1991.

[3] C.-T. Kuo, J.-T. Lim, S.M. Meerkov and E. Park, "Improvability Theory for Assembly Systems: Two Component - One Assembly Machine Case", *Math. Problems in Eng.*, vol. 3, pp. 95-171, 1997.

[4] S.-Y Chiang, *Bottlenecks in Production Systems with Markovian Machines: Theory and Applications*, Ph.D Thesis, Univ. of Michigan, Ann Arbor, MI, 1999.

[5] S.-Y. Chiang, C.-T. Kuo, J.-T. Lim and S.M. Meerkov, "Improvability Theory of Assembly Systems I: Problem Formulation and Performance Evaluation", *Math. Problems in Eng.*, vol. 6, pp. 321-357, 2000.

[6] E. Ignall and A. Silver, "The Output of a Two-stage System with Unreliable Machines and Limited Storage", *AIIE Trans.*, vol. 9, pp. 183-188, 1977.

[7] D. Mitra, "Stochastic Theory of a Fluid Model of Producers and Consumers Coupled by a Buffer", *Advanced Applied Probabilities*, vol. 20, pp. 646-676, 1988.

[8] T. Iyama and S. Ito, "The Maximum Production Rate for an Unbalanced Multi-server Flow Line System with Finite Buffer Storage", *Int. J. of Prod. Res.*, vol. 25, pp. 1157-1179, 1987.

[9] R. Alves, "Performance Evaluation of Series-parallel Systems", *J. of Manufacturing Operations Management*, vol. 3, pp. 224-250, 1990.

[10] M.H. Burman, *New Results in Flow Line Analysis*, Ph.D Thesis, MIT, Cambridge, MA, 1995.

[11] A. Patchong and D. Willaeys, "Modeling and Analysis of an Unreliable Flow Line Composed of Parallel-machine Stages", *IIE Trans.*, vol. 33, pp. 559-568, 2001.

[12] K.-C. Jeong and Y.-D. Kim, "An Approximation Method for Performance Analysis of Assembly/Disasembly Systems with Parallel-machine Stations", *IIE Trans.*, vol. 31, pp. 391-394, 1999.

[13] J.-T. Lim and S.M. Meerkov, "On Asymptotic Reliable Closed Serial Production Lines", *Control Engineering Practices*, vol. 1, pp. 147-152, 1993.

[14] Y. Frein, C. Commault and Y. Dallery, "Modeling and Analysis of Closed-Loop Production Lines with Unreliable Machines and Finite Buffers", *IIE Trans.*, vol. 28, pp. 545-554, 1996.

[15] S.B. Gershwin, N. Maggio, A. Matta, T. Tolio and L.M. Werner, "Analysis of Loop Networks by Decomposition", *Proc. of 3rd Aegean Int. Conf. on Design and Analysis of Manufacturing Systems*, pp. 239-248, Tinos Island, Greece, May. 2001.

[16] J. Li, "Overlapping Decomposition: A System-Theoretic Method for Modeling and Analysis of Complex Production Systems", *Technical Report*, General Motors Research & Development Center, 2003.

[17] S.-Y. Chiang, C.-T. Kuo and S.M. Meerkov, "DT-bottlenecks in Serial Production Lines: Theory and Application", *IEEE Trans. on Robotics and Automation*, vol. 16, pp. 567-580, 2000.

â€“Muaz A. Niazi, Complex Adaptive Systems Modeling 2016 4:3". Sayama's book is a very good instrument for students who want to read an introductory text on modeling and analysis of complex systems, and for instructors who need such a text in simple language for their complex systems courses and projects. The book offers a good introduction to the complex systems terminology and plenty of readily available examples with technical implementation details. Overall, Introduction to the Modeling and Analysis of Complex Systems offers a novel pedagogical approach to the teaching of complex systems, based on examples and library code that engage students in a tutorial-style learning adventure. MODELING AND ANALYSIS OF DYNAMIC SYSTEMS Third Edition EDITOR EDITORIAL ASSISTANT Steve Peterson MARKETING MANAGER Katherine Hnm11;cn SENIOR PRODUCTION EDITOR SENIOR DESIGNER Kevin Cover courtesy of NASA This book was set in Times Roman ancl bouncl Hamilton Press. This book was on acid-free paper. @ Copyright 2002 Â© John Wiley & Sons, Inc. All rights reserved. with modeling complex systems with a large number of variables. Abstract. Sayama, H Introduction to the Modeling and Analysis of Complex Systems Open SUNY. textbooks, Milne Library, State University of New York at Geneseo (2015). 485 pagesÂ and analysis of complex systems are described. Part II. Chapter 3 describes fundamental concepts of dynamical systems and phase spaces. Chapter 4 describes discrete time modeling using dïï¬€erence equations with a hands-on. approach. Chapter 5 focuses on the analysis of discrete-time models including the dis-. covery of equilibrium points, phase space visualization, and cobweb plots among other. topics. Business process modeling (BPM) in business process management and systems engineering is the activity of representing processes of an enterprise, so that the current business processes may be analyzed, improved, and automated. BPM is typically performed by business analysts, who provide expertise in the modeling discipline; by subject matter experts, who have specialized knowledge of the processes being modeled; or more commonly by a team comprising both. Alternatively, the process model can be... A theory for the modelling of complex and dynamic systems. SUBMITTED: June 2007 REVISED: August 2008 PUBLISHED: September 2008 EDITOR: B-C BjÃ¶rk. Wim Gielingh, Dr. Technical University of Delft email: wf@gielingh.nl.Â Systems are often modelled and depicted graphically by means of an inverted tree structure. The top (or root) of this inverted tree depicts the whole; the branches depict the parts; see also figure 1. (system).Â All knowledge created in a design process, but also in design analysis, work preparation and planning, is part of the 'imaginary world'. This imaginary world may become reality through production and/or construction.